97. *Congruences between Siegel Modular Forms of Degree Two*

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Introduction. We present some congruences between eigenvalues of Hecke operators on Siegel modular forms of degree two. In § 1, we give a congruence modulo 71^2 between two Siegel modular forms of degree two and weight 20 denoted here by $\chi_{20}^{(3)}$ and $[\Delta_{20}]$. This congruence seems to suggest that 71 will be an "exceptional prime" for $\chi_{20}^{(3)}$. In § 2, we give some other congruences. Some of them are reduced to the elliptic modular case by the recent results of Maass [5] and Andrianov [1]; Maass [5] proves, by using Shimura's theory of elliptic modular forms of half integral weight, that the Conjectures 1 and 2 of [4] hold (at least) for $k \leqq 20$. In § 3, we give a conjectural "interpretation" of these congruences.

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§ 1. A congruence. We follow the notations of [4] throughout this paper. Let $\chi_{20}^{(3)} = f_{20} - g_{20} + 595200 h_{20}$ with $f_{20} = 4\chi_{10}\varphi_4\varphi_6$, $g_{20} = 12\chi_{12}\varphi_4^2$, and $h_{20} = 48\chi_{10}^2$ as in § 5 of [4]. This is an eigen cusp form of degree two and weight 20. We denote by $[\Delta_{20}]$ the *eigen* modular form of degree two and weight 20 which is uniquely determined by $\Phi([\Delta_{20}]) = \Delta_{20}$, where $\Phi: M_{20}(\Gamma_2) \to M_{20}(\Gamma_1)$ is the Siegel Φ-operator. We prove the following congruence between $\chi_{20}^{(3)}$ and $[\Delta_{20}]$.

Theorem 1. $\lambda(m, \chi_{20}^{(3)}) \equiv \lambda(m, [\Delta_{20}]) \bmod 71^2$ *for all integers* $m \geqq 1$.

Proof. Let $e_{20} = 2^{-6} \cdot 3^{-3} \cdot (\varphi_4^5 - \varphi_4^2\varphi_6^2)$, then we have the following equality:

$$11[\Delta_{20}] + 38 \cdot 71^{-2} \chi_{20}^{(3)} = 11 e_{20} - 922 f_{20} - 614 g_{20} - 5030400 h_{20}.$$

This equality is proved as follows. Since $\Phi([\Delta_{20}]) = \Phi(e_{20}) = \Delta_{20}$ (hence $[\Delta_{20}]$