10. On Weak Evolution Operators with Constant Coefficients and Spreading of Wave Packets

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Some algebraic properties of a polynomial $P(\xi)$ are often determined if the corresponding partial differential equation $P(D)u=0$ has a nontrivial solution of special type. One of the important such results is the theorem due to F. John [4]. In that article he discussed the relation between the weak hyperbolicity of polynomials and the existence of solutions that represent the propagation of support with finite speed. L. Ehrenpreis, in his book [1, chap. IX], treated a similar problem even for non-kowalevskian operators.

In this note we shall state that we can generalize their results (at least in the single equation case), introducing the notions of weak evolution operators and wave packet spreading. These notions, in a way, extend those of weak hyperbolicity and support propagation.

1. Weak evolution operators. Let $P(\sigma,\xi)$ be a polynomial in $n+1$ variables $(\sigma,\xi)=(\sigma,\xi_1,\cdots,\xi_n)$ with complex coefficients of the form

$$P(\sigma,\xi)=\sigma^l+\sum_{j=1}^{l}a_j(\xi)\sigma^{l-j}, \tag{1}$$

for some integer $l\geqq 1$. Let us put $p=\operatorname{Max}_j(\deg a_j)/j$ and denote the homogeneous part of degree pj of a_j by a_j^0. Then the principal part of P is defined by

$$P^0(\sigma,\xi)=\sigma^l+\sum_{j=1}^{l}a_j^0(\xi)\sigma^{l-j}. \tag{2}$$

As usual we write $(D_t,D_x)=\dfrac{1}{i}\left(\dfrac{\partial}{\partial t},\dfrac{\partial}{\partial x_1},\cdots,\dfrac{\partial}{\partial x_n}\right)$ and define the partial differential operator $P(D_t,D_x)$ in $\boldsymbol{R}^{n+1}$. We owe the following definition, as well as the above one of P^0, to S. Mizohata [6].

Definition 1. Given a rational number $\alpha>0$, we shall call $P(D_t,D_x)$ the weak α-evolution operator with respect to the half space $H=\{(t,x)\,;\,t\geqq 0, x\in\boldsymbol{R}^n\}$ if either $\alpha>p$ or the following holds; $\alpha=p$ is an even integer and the imaginary parts of all the roots of $P^0(\sigma,\xi)=0$, $\xi\in\boldsymbol{R}^n$, are $\geqq 0$, or $\alpha=p$ is an odd integer and all those roots are real. We shall call P simply the weak α-evolution operator (resp. of S-type) if either $\alpha>p$, or $\alpha=p$ is an integer and real are all the roots of $P^0(\sigma,\xi)$

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