93. The Structure of the Albanese Map of an Algebraic Variety of Kodaira Dimension Zero

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In this paper we shall sketch an outline of a proof of the following

Main Theorem. *Let X be a non-singular projective algebraic variety defined over the complex number field C and let $\alpha: X \to A(X)$ be the Albanese map. We assume that $\kappa(X)=0$. Then α is a fiber space.*

A *fiber space* is a morphism of non-singular projective algebaric varieties which is surjective and has connected fibers.

Corollary. *If $\kappa(X)=0$, then we have $q(X) \underset{\text{def}}{=} \dim H^0(X, \Omega_X^1) \leqq \dim X$. Moreover, if the equality holds, then α is birational. Thus $\kappa(X)=0$ and $q(X)=\dim X$ give a characterization for an abelian variety up to birational equivalences.*

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The proof of Main Theorem is carried out along the standard program of classification theory of algebraic varieties (cf. [6]). The main point is the following "addition theorem".

Theorem 1. *Let $f: X \to Y$ be a fiber space and assume that $\kappa(X) \geqslant 0$ and $\kappa(Y)=\dim Y$. Then $\kappa(X)=\kappa(Y)+\kappa(F)$, where F is a general fiber.*

Beside this we have to know something about abelian varieties:

Theorem 2. *Let $f: X \to A$ be a generically finite morphism from a non-singular projective algebraic variety to an abelian variety. Then $\kappa(X) \geqslant 0$ and when we replace a birational model of X, if necessary, the Iitaka fibering $\Phi: X \to \bar{X}$ satisfies the following conditions:*

(1) *There is an abelian subvariety B of A and a general fiber of Φ is birationally equivalent to an etale cover of B.*

(2) *$\bar{X}$ is generically finite over A/B.*

(3) *$\kappa(\bar{X})=\dim \bar{X}=\kappa(X)$.*

The proof of Theorem 2 follows from Theorem "B_n" of [3]. Main