88. *The Range of Picard Dimensions*[*)]

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1. **Densities and Picard dimensions.** We will view the punctured unit disk $\Omega: 0<|z|<1$ as an *end* of $0<|z|\leqq+\infty$, a parabolic Riemann surface, so that the unit circle $|z|=1$ is the relative boundary $\partial\Omega$ of Ω and the origin $z=0$ is the single ideal boundary component $\delta\Omega$ of Ω. A *density* P on Ω is a nonnegative locally Hölder continuous function $P(z)$ on $\bar{\Omega}: 0<|z|\leqq 1$ which may or may not have a singularity at $\delta\Omega$. We denote by $PP(\Omega;\partial\Omega)$ the class of nonnegative solutions u of $\Delta u=Pu$ on Ω with vanishing boundary values on $\partial\Omega$. We also denote by $PP_1(\Omega;\partial\Omega)$ the subclass of $PP(\Omega;\partial\Omega)$ consisting of functions u with the normalization $u(a)=1$ for some fixed point a in Ω. We denote by ex. $PP_1(\Omega;\partial\Omega)$ the set of extreme points in the convex set $PP_1(\Omega;\partial\Omega)$. The cardinal number $\sharp(\text{ex.}\, PP_1(\Omega;\partial\Omega))$ of ex. $PP_1(\Omega;\partial\Omega)$ will be referred to as the *Picard dimension*, $\dim P$ in notation, of a density P at $\delta\Omega$:

$$\dim P=\sharp(\text{ex.}\, PP_1(\Omega;\partial\Omega)). \tag{1}$$

It is easily seen (cf. e.g. [7]) that $\dim P\geqq 1$ for any density P on Ω. A density P on Ω with $\dim P=1$ is said to satisfy the *Picard principle* at $\delta\Omega$.

2. **Problem and result.** We denote by $\mathscr{D}(\Omega)$ the class of densities on Ω. Consider the mapping $\dim: \mathscr{D}(\Omega)\to\{\text{cardinal numbers}\}$ defined by $P\mapsto \dim P$. We proposed to study the *range* $\dim\mathscr{D}(\Omega)=\{\dim P; P\in\mathscr{D}(\Omega)\}$ of the mapping dim in our former paper (cf. [5]). Virtually nothing has been known on $\dim\mathscr{D}(\Omega)$ except for the following simple fact (cf. [4], [6], [2]):

$$\dim P_\lambda=\begin{cases}1 & (\lambda\leqq 2)\\ \mathfrak{c} & (\lambda>2)\end{cases}$$

where P_λ is the density on Ω given by $P_\lambda(z)=|z|^{-\lambda}$ for real numbers λ and $\mathfrak{c}$ is the cardinal number of continuum. In view of this our *problem* is to determine whether the range $\dim\mathscr{D}(\Omega)$ contains cardinal numbers between 1 and $\mathfrak{c}$. Specifically we are interested in the question whether $\dim\mathscr{D}(\Omega)$ contains every countable cardinal numbers ξ, i.e. $\xi=n$, a positive integer, or $\xi=\mathfrak{a}$, the cardinal number of countably infinite set. The purpose of this note is to announce and also to give

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