84. *Evaluation of Peirce's Axiom on Intermediate Kripke Models and its Application*

By Tsutomu HOSOI
Department of Information Sciences,
Science University of Tokyo

(Communicated by Kunihiko KODAIRA, M. J. A., Nov. 12, 1979)

We deal with the intermediate propositional logics. We suppose familiarity with the intermediate logics. For those not mentioned here explicitly, we refer to our Survey [1].

The so-called Peirce's axiom P is $a \supset b \supset a \supset a$, where parentheses are omitted by assuming the association from the left. This axiom was modified by Nagata into a sequence of axioms as follows:

Definition 1. $P_1(a_0, a_1) = a_1 \supset a_0 \supset a_1 \supset a_1$,

$$P_n(a_0, \cdots, a_n) = a_n \supset P_{n-1}(a_0, \cdots, a_{n-1}) \supset a_n \supset a_n.$$

This sequence is often used in the study of intermediate logics as a strong and convenient tool. The evaluation of axioms from this sequence is sometimes treated in literatures but the treatment seems not to be complete.

Here we give a complete treatment of the evaluation and its application for the axiomatization of infinite models.

As models for intermediate propositional logics, we take up the so-called Kripke model, which was modified and renamed as POS model by Ono (see Ono [2]). Further, we treat only finite models.

Let M be a POS model (usually, with the minimum element). We define the condition $\mathrm{C}(W, \alpha)$ for the M-valuation W and the element α of M as follows:

Condition.

$\mathrm{C}(W, \alpha)$: *$W(a, \alpha) = \boldsymbol{f}$ and, for any $\beta > \alpha$, $W(a, \beta) = \boldsymbol{t}$.*
And there exists $\beta > \alpha$ such that $W(b, \beta) = \boldsymbol{f}$.

The main result of this note is the following

Theorem 2. *$W(P, \alpha) = \boldsymbol{f}$ if and only if there exists $\alpha_0 \geq \alpha$ such that* $\mathrm{C}(W, \alpha_0)$.

We prove the theorem through the two lemmas as below.

Lemma 3. *$\mathrm{C}(W, \alpha_0)$ implies $W(P, \alpha_0) = \boldsymbol{f}$.*

Proof. By the hypothesis, we have $W(a, \beta) = \boldsymbol{t}$ for any $\beta > \alpha_0$. Further, there exists $\beta > \alpha_0$ such that $W(b, \beta) = \boldsymbol{f}$. Hence we have $W(a \supset b, \alpha_0) = \boldsymbol{f}$. So we have, for any $\gamma \geq \alpha_0$, $W(a \supset b, \gamma) = \boldsymbol{f}$ or $W(a, \gamma) = \boldsymbol{t}$. Hence we have $W(a \supset b \supset a, \alpha_0) = \boldsymbol{t}$. Finally, from $W(a \supset b \supset a, \alpha_0)$