79. *The Initial Value Problem for the Equations of Motion of Compressible Viscous and Heat-Conductive Fluids*

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§ 1. Introduction and theorem. The motion of the general isotropic Newtonian fluids are described by the five conservation laws:

$$(1.1)\quad \begin{cases} \rho_t+(\rho u^j)_{x_j}=0 \\ u^i_t+u^j u^i_{x_j}+\dfrac{1}{\rho}p_{x_i}=\dfrac{1}{\rho}\{(\mu(u^i_{x_j}+u^j_{x_i}))_{x_j}+(\mu' u^j_{x_j})_{x_i}\}, \qquad i=1,2,3 \\ \theta_t+u^j\theta_{x_j}+\dfrac{\theta p_\theta}{\rho c}u^j_{x_j}=\dfrac{1}{\rho c}\{(\kappa\theta_{x_j})_{x_j}+\Psi\}, \end{cases}$$

where ρ: density, $u=(u^1,u^2,u^3)$: velocity, θ: absolute temperature, $p=p(\rho,\theta)$: pressure, $\mu=\mu(\rho,\theta)$: viscosity coefficient, $\mu'=\mu'(\rho,\theta)$: second viscosity coefficient, $\kappa=\kappa(\rho,\theta)$: coefficient of heat conduction, $c=c(\rho,\theta)$: heat capacity at constant volume and $\Psi=\frac{\mu}{2}(u^j_{x_k}+u^k_{x_j})^2+\mu'(u^j_{x_j})^2$: dissipation function. We consider the initial value problem for (1.1) with the initial data

(1.2) $(\rho,u,\theta)(0,x)=(\rho_0,u_0,\theta_0)(x),\ x\in R^3$.

We seek the solutions in a neighbourhood of a constant state $(\rho,u,\theta)=(\bar\rho,0,\bar\theta)$, where $\bar\rho$, $\bar\theta$ are any positive constants. Thus we assume a natural condition on the system (1.1) of hyperbolic-parabolic type throughout this paper that

(i) p, c, μ, μ' and κ are smooth functions in $\mathcal{O}=\{(\rho,u,\theta):|\rho-\bar\rho|,|u|,|\theta-\bar\theta|<\bar\varepsilon\}$.

(ii) $\partial p/\partial\rho$, $\partial p/\partial\theta>0$, $c,\mu,\kappa>0$ and $\mu'+\frac{2}{3}\mu\ge 0$ in $\mathcal{O}$,

where $\bar\varepsilon<\min\{\bar\rho,\bar\theta\}$.

First rewrite the system (1.1) by the change of the unknown and known variables as follows: $\rho\to\bar\rho+\rho$, $u\to u$, $\theta\to\bar\theta+\theta$, $p(\bar\rho+\rho,\bar\theta+\theta)\to p(\rho,\theta)$, $\mu(\bar\rho+\rho,u,\bar\theta+\theta)\to\mu(\rho,u,\theta)$ and so on.

$$(1.3)\quad \begin{cases} L^0(\rho,u)\equiv\rho_t+(\bar\rho+\rho)u^j_{x_j}+u^j\rho_{x_j}=0 \\ L^i(u)\equiv u^i_t-\tilde\mu u^i_{x_jx_j}-(\tilde\mu+\tilde\mu')u^j_{x_ix_j}=G^i, \qquad i=1,2,3 \\ L^4(\theta)\equiv\theta_t-\tilde\kappa\theta_{x_jx_j}=G^4, \end{cases}$$

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