77. *Remarks on Hadamard's Variation of Eigenvalues of the Laplacian*

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§1. **Introduction.** The study in this note is a continuation of our previous paper [6]. Let Ω be a bounded domain in $\mathbf{R}^n$ $(n\geq 2)$ with C^∞ boundary γ. Let $\rho(x)$ be a smooth function on and ν_x be the exterior unit normal vector at $x\in\gamma$. For any sufficiently small $\varepsilon\geq 0$, let Ω_ε be the bounded domain whose boundary γ_ε is defined by $\gamma_\varepsilon=\{x+\varepsilon\rho(x)\nu_x\,;\,x\in\gamma\}$. Let $U_\varepsilon(x,y,t)$ be the Green kernel of the heat equation in Ω_ε with the Dirichlet boundary condition on γ_ε. Let $T_r(t\,;\varepsilon)$ be the trace of U_ε on Ω_ε. When t tends to zero, we have the asymptotic expansion $T_r(t\,;\varepsilon)\sim\sum_{j=0}^{\infty}a_{n-j}(\varepsilon)(\sqrt{t})^{-n+j}$ which was given by Minakshisundarum-Pleijel [5]. In [6], the author gave the asymptotic expansion $\delta T_r(t)\sim\sum_{j=0}^{\infty}b_{n-j}(\sqrt{t})^{-n+j}$ near $t=0$ of the variational term $\delta T_r(t)$ of the trace which was defined by $\delta T_r(t)=\lim_{\varepsilon\to 0}\varepsilon^{-1}(T_r(t\,;\varepsilon)-T_r(t\,;0))$. We proposed the following problem $(\mathrm{E})_k^n$ in [6] and gave an affirmative answer for the case $k=0$.

Problem $(\mathrm{E})_k^n$. *Can we say that the following is valid?*

$$(\mathrm{E})_k^n \qquad b_{n-k}=\lim_{\varepsilon\to 0}\varepsilon^{-1}(a_{n-k}(\varepsilon)-a_{n-k}(0)).$$

In this paper, we shall prove the following

Theorem 1. $(\mathrm{E})_1^n$ *is valid for any* $n\geq 2$.

The aims of this note are verification of Theorem 1 and an application of Theorem 1 to some eigenvalue problem which will be stated in this section.

We now mention the following

Problem (Q). *Characterize the bounded domain* Ω *with smooth boundary* γ *having the following property.*

(I) *For any* $\rho(z)\in C^\infty(\gamma)$ *such that* $\int_\gamma\rho(z)d\sigma_z=0$, *we have* $\delta\lambda_1=0$, *where* $\delta\lambda_1$ *is the variational term of the first eigenvalue* $\lambda_1<0$ *of the Laplacian with the Dirichlet condition. Here* $d\sigma_z$ *denotes the surface element of* γ *at* z.

The condition $\int_\gamma\rho(z)d\sigma_z=0$ means that the perturbation of domain we considered preserves the volume of domains infinitesimally. We