76. *Poisson Transformations on Affine Symmetric Spaces*[*)]

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1. Introduction. Let G be a connected real semisimple Lie group with finite center, σ any involutive analytic automorphism of G, and H any closed subgroup which lies between the totality G^σ of fixed points of σ and the identity component of G^σ. Then the homogeneous space G/H is an *affine symmetric space*. It is known that any eigenfunction of all invariant differential operators on a Riemannian symmetric space of the noncompact type can be represented by the Poisson integral of a hyperfunction on its maximal boundary, which was conjectured by Helgason [2] and completely solved by [3]. In this note we define a generalization of the Poisson integral on G/H and extend the result in [3] to the case of G/H. For example, by the involution $(g, g') \mapsto (g', g)$ of $G \times G$, the group G itself can be regarded as an affine symmetric space and then our result gives integral representations of simultaneous eigenfunctions of biinvariant differential operators on G. If G/H satisfies some conditions, this problem was studied by [6] (cf. also [5]). An extended version of this note is to appear later.

2. Preliminary results. We fix a Cartan involution θ of G commuting with σ (cf. [1] for the existence of θ) and also denote by σ and θ the corresponding involutions of the Lie algebra $\mathfrak{g}$ of G. Let $\mathfrak{g}=\mathfrak{k}+\mathfrak{p}$ (resp. $\mathfrak{g}=\mathfrak{h}+\mathfrak{q}$) be the decomposition of $\mathfrak{g}$ into $+1$ and -1 eigenspaces for θ (resp. σ). Let $\mathfrak{a}$ be a maximal abelian subspace of $\mathfrak{p}\cap\mathfrak{q}$, $\mathfrak{a}_\mathfrak{p}$ a maximal abelian subspace of $\mathfrak{p}$ containing $\mathfrak{a}$, and $\tilde{\mathfrak{j}}$ a Cartan subalgebra of $\mathfrak{g}$ containing both $\mathfrak{a}_\mathfrak{p}$ and a maximal abelian subspace of $\mathfrak{m}\cap\mathfrak{q}$, where $\mathfrak{m}$ denotes the centralizer of $\mathfrak{a}_\mathfrak{p}$ in $\mathfrak{k}$. Furthermore we put $\mathfrak{j}=\tilde{\mathfrak{j}}\cap\mathfrak{q}$ and $\mathfrak{t}=\mathfrak{j}\cap\mathfrak{k}$. For a linear subspace $\mathfrak{b}$ of $\mathfrak{g}$, $\mathfrak{b}_c$ denotes the complexification of $\mathfrak{b}$. If $\mathfrak{b}$ is a subalgebra, $U(\mathfrak{b})$ denotes the universal enveloping algebra of $\mathfrak{b}_c$. Let Ad (resp. ad) denote the adjoint representation of G (resp. $\mathfrak{g}_c$) on $\mathfrak{g}_c$ or $U(\mathfrak{g})$. For a linear subspace $\tilde{\mathfrak{a}}$ of $\tilde{\mathfrak{j}}$, $\tilde{\mathfrak{a}}^*$ denotes the dual space of $\tilde{\mathfrak{a}}$ and $\tilde{\mathfrak{a}}_c^*$ the complexification of $\tilde{\mathfrak{a}}^*$. Then we put $\mathfrak{g}_c(\tilde{\mathfrak{a}};\lambda)=\{X\in\mathfrak{g}_c;\ ad(Y)X=\lambda(X) \text{ for all } Y\in\tilde{\mathfrak{a}}\}$ for any λ in $\tilde{\mathfrak{a}}_c^*$ and moreover $\Sigma(\tilde{\mathfrak{a}})=\{\lambda\in\tilde{\mathfrak{a}}_c^*-\{0\};\ \mathfrak{g}_c(\tilde{\mathfrak{a}};\lambda)\neq\{0\}\}$. By the Killing form $\langle\ ,\ \rangle$ of the complex Lie algebra $\mathfrak{g}_c$, we identify $\mathfrak{j}_c^*$ and $\mathfrak{j}_c$, and therefore $\tilde{\mathfrak{a}}_c^*$ is identified with a subspace of $\mathfrak{j}_c^*$. Let K denote the analytic subgroup of G cor-

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