68. On the Pseudo-Parabolic Regularization of the Generalized Kortweg-de Vries Equation

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1. Introduction. This note is concerned with the initial-boundary value problem:

(1) $$u_t+(\phi(u))_x+u_{xxx}-\varepsilon u_{txx}=0, \qquad t\in R,\ x\in(0,1),$$

(2) $$u(0,x)=g(x), \qquad x\in(0,1),$$

(3) $$u(t,0)=u(t,1), \qquad t\in R,$$

where $\varepsilon>0$, ϕ is a function of class $C^\infty(R)$ satisfying $\phi(0)=0$ and g is a given initial function satisfying $g(0)=g(1)$.

The pseudo-parabolic equation (1) is understood to be a generalization of model equations for long water waves of small amplitude (see for instance [1]). The equation (1) is also regarded as a regularization of the generalized Kortweg-de Vries equation

(4) $$u_t+(\phi(u))_x+u_{xxx}=0.$$

For the parabolic regularizations of the generalized KdV equation, see [4].

Here we treat the initial-boundary value problem (1)–(3) from the viewpoint of the semigroup theory and describe the properties of solutions of the problem in terms of nonlinear group in a Hilbert space.

2. Theorem. We denote by $\|\cdot\|$ the norm of the Lebesgue space $L^2(0,1)$. For each positive integer m, we write V^m for the closed subspace of the Sobolev space $H^m(0,1)$ defined by

$$V^m=\{v\in H^m(0,1)\,;\ v^{(l)}(0)=v^{(l)}(1),\ 0\leqq l\leqq m-1\}.$$

We also denote by D the differential operator d/dx from $H^1(0,1)$ into $L^2(0,1)$, i.e., D is defined by $Dv=v'$ for $v\in H^1(0,1)$.

Now we define a linear operator L_ε from V^2 into V^1 by

$$L_\varepsilon v=\frac{1}{\varepsilon}Dv \qquad \text{for } v\in V^2,$$

and a nonlinear operator F_ε on V^1 by

$$[F_\varepsilon v](x)=\int_0^1 K_\varepsilon(x,\xi)\left\{\phi(v(\xi))+\frac{1}{\varepsilon}v(\xi)\right\}d\xi$$

for $v\in V^1$ and $x\in[0,1]$, where

$$K_\varepsilon(x,\xi)=\frac{\operatorname{sgn}(x-\xi)}{2(1-e)\varepsilon}\left\{\exp\left(\frac{|x-\xi|}{\sqrt{\varepsilon}}\right)-\exp\left(1-\frac{|x-\xi|}{\sqrt{\varepsilon}}\right)\right\} \quad \text{for } x,\xi\in[0,1].$$

Note that $w\equiv F_\varepsilon v$ gives a unique solution of the boundary value problem