61. On Some Periodic 4-Transitive Permutation Groups

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1. Introduction. In [2], O. H. Kegel determined the locally finite Zassenhaus groups with some additional conditions. By making use of some ideas in the proofs of M. Hall [1] and V. P. Shunkov [4], we shall prove the following theorem allied to Kegel's result.

Theorem. *Let G be a periodic 4-transitive permutation group on a set Ω ($|\Omega|\leqq\infty$). If $G_{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5}=1$ for any distinct five points $\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5$ of Ω, then G is a finite group and is isomorphic to one of the following groups: $S_4, S_5, S_6, A_6, A_7, M_{11}$ or M_{12}.*

Definitions. Let G be a group. G is called a periodic group if every element of G has finite order. G is called a locally finite group if every finite subset of G generates a finite group. G is called a Frobenius group if G contains a proper subgroup H such that $g^{-1}Hg\cap H=1$ for all $g\in G-H$. Such a subgroup H of the Frobenius group G is called a Frobenius complement of G.

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2. Proof of Theorem. In the first place, we prove the following

Lemma. *Let G be a periodic Frobenius group and H a Frobenius complement of G. Then H contains at most one involution.*

Proof. Suppose, by way of contradiction, that H contains two involutions i and j. Let g be an involution in $G-H$. First we show that there exists an involution y in $G-H$ such that $y^{-1}iy=g$. If $|ig|$ (=the order of ig) is even, then we have $ia=ai$ and $ga=ag$ for the involution a in $\langle ig\rangle$. Therefore we have $a\in C_G(i)\subseteq H$, and so we have $g\in C_G(a)\subseteq H$, a contradiction. Hence there exists an element x in $\langle ig\rangle$ such that $x^{-1}ix=g$, because $|ig|$ is odd. Set $ix=y$. Then y is an involution in $G-H$ such that $y^{-1}iy=g$. Similarly, there exists an involution z in $G-H$ such that $z^{-1}jz=g$. Since yz normalizes H and $y^{-1}Hy$ ($=z^{-1}Hz$), we have $yz=1$. Hence we have $i=j$, a contradiction.

Proof of Theorem. Let G be a permutation group satisfying the assumption of Theorem. If G is a finite group, then we know that G is isomorphic to $S_4, S_5, S_6, A_6, A_7, M_{11}$ or M_{12} (cf. [1], [3]). From now on, we shall assume that G is an infinite periodic group and $|\Omega|=\infty$, and prove eventually that this leads to a contradiction. We may assume that $\{1,2,3,\cdots\}\subseteq\Omega$.