# 51. *Construction of Complex Structures on Open Manifolds**)

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**1. Introduction.** In 1951, in his book [6] N. Steenrod conjectured, "it seems highly unlikely that every almost complex manifold has a complex analytic structure".

In [7] Van de Ven showed the existence of a compact almost complex manifold of dimension 4 which does not admit any complex structure.

Recently S.-T. Yau [8] and N. Brotherton [2] have shown some examples of compact parallelizable manifolds of dimension 4 which do not admit any complex structure.

On the other hand, in [3] M. Gromov has shown a method to obtain complex structures on a special almost complex manifold. As a corollary, he has shown that on an open manifold of dimension 4, any almost complex structure is homotopic to a complex one.

In this note we shall improve a little on Gromov's result on the construction of complex structures on open manifolds. As a corollary we shall prove that on an open 6-dimensional manifold, any almost complex structure is homotopic to a complex one.

We study this problem within the frame work of A. Haefliger [4], [5] which permits one to view the problem as a lifting problem in homotopy theory.

The interest of Dr. K. Nakajima in the integrability of almost complex structures stimulated the appearance of the present note.

**2. Preliminaries.** We now give a brief recall on Haefliger's work [4], [5] that are needed here. Let $\Gamma_q^c$ denote the topological groupoid of germs of local complex analytic automorphisms of $C^q$, and let $B\Gamma_q^c$ denote a classifying space for $\Gamma_q^c$-structures. The differential induces a continuous homomorphisms

$$\nu : \Gamma_q^c \to GL(q, C),$$

hence also a continuous map

$$\nu : B\Gamma_q^c \to BGL(q, C).$$

We convert this map to a fibration and write $F\Gamma_q^c$ for the homotopy fibre. Consider the following diagram:

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*) Dedicated to Professor A. Komatu for his 70th birthday.