49. On the Boundary Behavior of Taylor Series of Regular Functions of Some Classes in the Unit Circle

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1. Introduction. In his previous papers ([3], [4]), the author introduced (C, k, α)-summation, by means of which Taylor series of the regular function of bounded type in $|z|<1$ can be summable on $|z|=1$. In this note, for the class wider than bounded type, he studies the convergence, the almost everywhere convergence and the mean convergence of this summation.

2. Statement of results. For the sake of completeness, we recall the definition of (C, k, α)-summation. Let $f(z)$ be a regular function in $|z|<1$:

$$f(z)=\sum_{n=0}^{\infty} a_n z^n.$$

For two constants $k, \alpha (k>-1, \alpha>O)$, we put

$$\frac{1}{(1-z)^{k+1}}\cdot \exp\left(\frac{\alpha}{1-z}\right)=\sum_{n=0}^{\infty} b_n(k,\alpha)z^n,$$

where

$$(1)\ b_n(k,\alpha)>O,\ (2)\ b_n(k,\alpha)\sim\frac{\exp(\alpha/2+2\sqrt{\alpha n})}{2\sqrt{\pi}\,\alpha^{1/4+k/2}n^{1/4-k/2}} \text{ as } n\to\infty,$$

and let

$$\frac{1}{(1-z)^{k+1}}\cdot \exp\left(\frac{\alpha}{1-z}\right)\sum_{n=0}^{\infty} a_n e^{in\theta}z^n=\sum_{n=0}^{\infty} S_n(k,\alpha,e^{i\theta})\cdot z^n.$$

If $C_n(k,\alpha,e^{i\theta})=S_n(k,\alpha,e^{i\theta})/b_n(k,\alpha)\to s$ as $n\to\infty$, we say that the series $\sum_{n=0}^{\infty} a_n e^{in\theta}$ is summable (C, k, α) to s.

Our Theorem 1 reads as follows.

Theorem 1. *Let $f(z)=\sum_{n=0}^{\infty} a_n z^n$ be a regular function in $|z|<1$ such that*

$$(2.1)\qquad \overline{\lim_{r\to 1}}\,(1-r)\cdot\log^+ M(r)=\delta<+\infty,$$

where $M(r)=\max_{|z|=r}|f(z)|$. Then the following propositions hold:

(A) *If $f(z)$ has the finite angular limit $f(e^{i\theta})$ at $z=e^{i\theta}$, then for any $\alpha>\delta$, $\sum_{n=0}^{\infty} a_n e^{in\theta}$ is summable (C, k, α) to $f(e^{i\theta})$.*

$$\text{(B)}\qquad \rho^n\cdot\int_{-\pi}^{\pi}|C_n(k,\alpha,e^{i\theta})-f(\rho e^{i\theta})|\,d\theta=o(1) \text{ as } n\to\infty,$$

where $\rho=1-\sqrt{\alpha/n}$, $\alpha>\delta$.