40. *Probabilistic Construction of the Solution of Some Higher Order Parabolic Differential Equation*

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§ 1. **Introduction.** The purpose of this note is to give a probabilistic solution of higher order partial differential equations of some specific type. We first recall that the solution of the heat equation

(1) $\dfrac{\partial u}{\partial t}=\dfrac{1}{2}\left(\dfrac{\partial}{\partial x}\right)^2 u,\ (t,x)\in(0,\infty)\times \boldsymbol{R}^1$

(2) $u(0,x)=u_0(x)$

is expressed in terms of a Brownian motion $\{\boldsymbol{B}_t\}_{t\geqq 0}$ in the form

(3) $u(t,x)=\boldsymbol{E}[u_0(x+\boldsymbol{B}_t)]$,

where $\boldsymbol{E}$ means the expectation. The key formula to prove that (3) satisfies (1) is $(d\boldsymbol{B}_t)^2=dt$. Being inspired by this formula, we take another Brownian motion w_t to expect that a formal formula $(d\boldsymbol{B}_{w_t})^4=dt$ would hold, so that the process $\boldsymbol{B}_{w_t}$ is related to the operator $\left(\dfrac{\partial}{\partial x}\right)^4$. More precisely, our problem is to express the solution of equation

(4) $\dfrac{\partial u}{\partial t}=\left(\dfrac{\partial}{\partial x}\right)^4 u$

in a similar form to (3) by using the process $\boldsymbol{B}_{w_t}$. However, $\boldsymbol{B}_{w_t}$ can not be viewed as a motion of some particle since the Brownian motion $\boldsymbol{B}_t$ as a diffusion process with generator $\dfrac{1}{2}\left(\dfrac{\partial}{\partial x}\right)^2$ is defined only for $t\geqq 0$, while w_t can take negative values. To overcome this difficulty, we need some trick as is illustrated in what follows.

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§ 2. **Simple case.** First we discuss a simple equation

(5) $\dfrac{\partial u}{\partial t}=\dfrac{1}{8}\left(\dfrac{\partial}{\partial x}\right)^4 u,\ (t,x)\in(0,\infty)\times \boldsymbol{R}^1.$

Let $\{\bar{\boldsymbol{B}}_t\}_{t\in \boldsymbol{R}^1}$ be a complex-valued stochastic process given by

$$\bar{\boldsymbol{B}}_t=\begin{cases}\boldsymbol{B}_t, & t\geqq 0,\\ i\boldsymbol{B}_{-t}, & t\leqq 0.\end{cases}$$

Denote by $\mathscr{D}_1$ the class of real-valued functions $f(x)$ defined on $\boldsymbol{R}^1$ which are extensible to entire functions $\bar{f}(z)$ on $\boldsymbol{C}^1$ satisfying the following conditions (6) and (7).