

35. On Plane Rational Curves

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1. The logarithmic Kodaira dimension $\bar{\kappa}(V)$ of an algebraic variety V , introduced by S. Iitaka [1], plays an important role in the study of algebraic varieties. In this note, we announce some results concerning the logarithmic Kodaira dimension $\bar{\kappa}(\mathbf{P}^2 - C)$, where C is an irreducible curve on \mathbf{P}^2 . Here we consider only rational curves C which satisfy

(A) C has one singular point,

or

(B) C has two singular points, which are cusps.

Any singular point P of C is said to be a *cusps*, if $\mu^{-1}(P)$ is one point, where $\mu: C^* \rightarrow C$ is the reduction of singularities. By the recent result of I. Wakabayashi [3], if the curve C does not satisfy the above conditions, then $\bar{\kappa}(\mathbf{P}^2 - C) = 2$ or C is a non-singular curve with degree ≤ 3 . Details will appear elsewhere. The author would like to express his hearty thanks to Prof. S. Iitaka for suggesting this problem and giving valuable advice.

2. In this note, we use the following convention. Let (X_0, X_1, X_2) denote the system of homogeneous coordinates on \mathbf{P}^2 and $x = X_1/X_0$, $y = X_2/X_0$. Since we do not consider $V_+(X_0)$, we say that the irreducible curve $V_+(X_0^n f(X_1/X_0, X_2/X_0))$ is defined by f , where f is an irreducible polynomial of degree n . Moreover let C denote a rational curve of degree n on \mathbf{P}^2 . In the case of (A), let P be the singular point of C , and put $e = \text{mult}_P(C)$. Denote by N the number of points of $\mu^{-1}(P)$. In the case of (B), let P, Q denote two singular points of C , and (e_1, \dots, e_p) [resp. (m_1, \dots, m_q)] the sequence of the multiplicities of all the (infinitely near) singular points of P [resp. Q].

3. Put $k = e$ in the case of (A), and $k = \max\{e_p, m_q\}$ in the case of (B), respectively. Then we have the following result.

Proposition 1. *In both cases, if $n \geq 3k$, then $\bar{\kappa}(\mathbf{P}^2 - C) = 2$.*

In the case of (B), define

$$M = \sum_{i=1}^p (e_i - 1) + \sum_{j=1}^q (m_j - 1) + d + 6 - 3n,$$

where $d = \min\{e_p, m_q\}$. Then

Proposition 2. *If $M > 0$, then $\bar{\kappa}(\mathbf{P}^2 - C) = 2$.*

However, it seems difficult to construct rational curves satisfying