33. On the Spectra of Laplace Operator on $\Lambda^*(S^n)$

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(Communicated by Kunihiko KODAIRA, M. J. A., April 12, 1979)

0. Let Δ be the Laplace operator acting on the space $\Lambda^*(S^n)$ of differential forms on the standard sphere S^n. A. Ikeda and Y. Taniguchi [1] and B. L. Beers and R. S. Millman [2] regarded Δ as the Casimir operator and determined its eigenvalues and multiplicities by using the representation theory.

On the other hand, S. Gallot and D. Meyer [4] tried to determine them by direct computations using harmonic homogeneous forms. But the result for multiplicities contains some errors. In this paper we show the complete result by an elementary method not using the representation theory.

1. Let D be the connection of $\boldsymbol{R}^{n+1}$ and ∇ the connection of S^n induced by the inclusion map ι from S^n into $\boldsymbol{R}^{n+1}$, where we use the canonical metrics. Then, for local vector fields X and Y on S^n, we know

$$D_X Y = \nabla_X Y - \langle X, Y\rangle X_{n+1}$$

and

$$D_X X_{n+1} = X,$$

where X_{n+1} is the locally extended vector field in $\boldsymbol{R}^{n+1}$ from the normal vector of norm 1 at each point of S^n by parallel transportation along the ray issuing from the origin, and X and Y are locally extended in $\boldsymbol{R}^{n+1}$.

Hereafter we extend the local vector field X on S^n so as to satisfy $[X, X_{n+1}]=0$. Also note that $D_{X_{n+1}}X_{n+1}=0$. Now denote by d_0, δ_0 and $\bar{\Delta}$ respectively the differential, its codifferential and Laplace operator on the space $\Lambda^p(\boldsymbol{R}^{n+1})$ of differential p-forms on $\boldsymbol{R}^{n+1}$ associated to D. Then, for any closed p-form α on $\boldsymbol{R}^{n+1}$, we have

$$\begin{aligned}&[\Delta(\iota^*\alpha)-\iota^*(\bar{\Delta}\alpha)]_x(i_1, \cdots, i_p)\\ &\quad = X_{n+1}|_x [X_{n+1}\alpha(i_1, \cdots, i_p)] + (n-2p+2)X_{n+1}|_x\, \alpha(i_1, \cdots, i_p)\end{aligned}$$

at any $x \in S^n$, where $\alpha(i_1, \cdots, i_p)=\alpha(X_{i_1}, \cdots, X_{i_p})$. (Cf. [4].)

Moreover, if α is a harmonic homogeneous p-form of degree k on $\boldsymbol{R}^{n+1}$, we have

$$\Delta(\iota^*\alpha)_x(i_1, \cdots, i_p) = (k+p)(n-p+k+1)\alpha_x(i_1, \cdots, i_p)$$

at any $x \in S^n$. (Cf. [4].)

Let H^p_k be the set of all coclosed harmonic homogeneous p-forms of degree k on $\boldsymbol{R}^{n+1}$ and let V^p_λ denote the subspace of $\Lambda^p(S^n)$ consisting of eigenforms associated to each eigenvalue λ of Δ. Since

$$\iota^* : \sum_{k\geqq 0} H^p_k \longrightarrow \Lambda^p(S^n)$$