26. *The Hodge Conjecture and the Tate Conjecture for Fermat Varieties*

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Throughout the paper, $X_m^n(p)$ will denote the Fermat variety of dimension n and of degree m in characteristic p ($p=0$ or a prime number not dividing m), defined by the equation

(1) $$x_0^m + x_1^m + \cdots + x_{n+1}^m = 0.$$

The purpose of this note is to report our results on the Hodge Conjecture for $X_m^n(0)$ and the Tate Conjecture for $X_m^n(p)$, $p>0$. By means of the inductive structure of $X_m^n(p)$ with respect to n ([3, § 1]), we can reduce the proof of these conjectures to the verification of certain purely arithmetic conditions on m, n and p. After formulating the condition in § 1, we state the main results in §§ 2 and 3. We give the brief sketch of the proof in § 4.

Detailed accounts will be published elsewhere.

§ 1. The arithmetic condition. Fix $m>1$, and let H be a cyclic subgroup of order f of $(\boldsymbol{Z}/m)^\times$. We consider the following system of linear Diophantine equations in $x_1, \cdots, x_{m-1}$ and y

(2) $$\sum_{\nu=1}^{m-1} \sum_{u \in H} \langle tu\nu \rangle x_\nu = fmy \qquad \text{for all } t \in (\boldsymbol{Z}/m)^\times,$$

where, for $a \in \boldsymbol{Z}/m - \{0\}$, $\langle a \rangle$ denotes the representative of a between 1 and $m-1$. Let $M_m(H)$ denote the additive semigroup of non-negative integer solutions $(x_1, \cdots, x_{m-1}; y)$ of (2) satisfying moreover the following congruence:

(3) $$\sum_{\nu=1}^{m-1} \nu x_\nu \equiv 0 \qquad (\text{mod } m).$$

For an element $\xi = (x_1, \cdots, x_{m-1}; y)$ of $M_m(H)$, we call y the length of ξ and write $y = \|\xi\|$. (We exclude the trivial solution $(0, \cdots, 0; 0)$.) If H' is a cyclic subgroup of H, then $M_m(H')$ is contained in $M_m(H)$; in particular, setting $M_m = M_m(\{1\})$, we have $M_m \subset M_m(H)$ for any H. There are exactly $[m/2]$ elements of length 1 in $M_m(H)$ and they are all contained in M_m.

Definition. Let $\xi = (x_1, \cdots, x_{m-1}; y) \in M_m(H)$. Then

(i) ξ is *decomposable* if $\xi = \xi' + \xi''$ for some $\xi', \xi'' \in M_m(H)$; otherwise ξ is called *indecomposable*.

(ii) ξ is *quasi-decomposable* if there exists $\eta \in M_m(H)$ with $\|\eta\| \le 2$ such that $\xi + \eta = \xi' + \xi''$ for some $\xi', \xi'' \in M_m(H)$ with $\|\xi'\|, \|\xi''\| < \|\xi\|$.

(iii) ξ is *semi-decomposable* if there exist non-negative integer