24. Experiments Concerning the Distribution of Squarefree Numbers

By Minoru TANAKA
Department of Mathematics, Faculty of Science,
Gakushuin University, Toshima-ku, Tokyo

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Let $Q(x)$ denote the number of squarefree integers not exceeding x. In this note some numerical results concerning $Q(x)$ obtained by the author will be reported. Before listing the results, we shall briefly refer to the theoretical property of $Q(x)$. Put for brevity

$$R(x)=Q(x)-\frac{6}{\pi^2}x.$$

As is well known, it can elementarily be proved that

$$R(x)=O(\sqrt{x}).$$

(cf. [1, p. 269]; [2, p. 582]; [3, p. 198]) Also, using the prime number theorem, or the fact that the Riemann zeta function $\zeta(s)$ has no zeros on the line $\sigma=1$, we can prove that

$$R(x)=o(\sqrt{x}).$$

(cf. [2, § 162, p. 606])

On the other hand, by similar way as in [2], Fünftes Buch, Zwanzigster Teil, we can prove that

$$\liminf_{x\to\infty} x^{-1/4}R(x)<0, \qquad \limsup_{x\to\infty} x^{-1/4}R(x)>0,$$

so that $R(x)$ changes its sign infinitely often as x tends to infinity.

Here we list some results selected from the large amount of computer output.

The first line of Table I means that approximately $R(100)=.2$, $R(200)=.4$, $R(300)=.6$, $R(400)=-.1$, $R(500)=2.0$, $\cdots$. We omitted the figure below the first place of decimals for each $R(x)$.

The formula

$$Q(x)=\sum_{n\le\sqrt{x}}\mu(n)\left[\frac{x}{n^2}\right]$$

was used. (cf. [1, p. 269]; [2, p. 581]) The computation was carried out at the Computer Center of Gakushuin University.

As is seen from the tables, the value of $R(x)$ frequently changes its sign. This phenomenon is in conformance with the above-mentioned theoretical result. Also it would be worth while noting that the absolute value of $R(x)$ is astonishingly small compared with x.