16. *Generalized Multiple Wiener Integrals*

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§1. **Introduction.** The analysis of nonlinear functionals of Brownian motion $\{B(t)\}$, which we call simply *Brownian functionals*, can be expressed in terms of white noise $\{\dot{B}(t)\}$, $\dot{B}(t)=dB(t)/dt$. We are specifically interested in the so-called *causal calculus* where the propagation of time is taken into account. Intuitively speaking, $\{\dot{B}(t)\}$ may be taken to be a coordinate system of the basic space on which Brownian functionals are defined. At the same time, $\{\dot{B}(t)\}$ could be thought of as a system of variables of Brownian functionals. With this system the passage of time, say by h, can be represented explicitly as $\dot{B}(t)\to\dot{B}(t+h)$. In order to carry out the causal calculus we have naturally been led to the concept of generalized Brownian functionals ([3]). There we were inspired by P. Lévy's work [1] on functional analysis.

The purpose of this note is to discuss those generalized Brownian functionals by expressing them as generalized multiple Wiener integrals with respect to the generalized random measures formed from polynomials in the $\dot{B}(t)$'s. There we can see that our expression of generalized Brownian functionals is most fitting for the causal calculus in question.

§2. **Known results.** Brownian functionals with finite variance can be expressed in terms of white noise and realized as members of $(L^2)=L^2(\mathcal{S}^*,\mu)$, where $\mathcal{S}^*$ is the dual space of the Schwartz space $\mathcal{S}$ on R and μ is the probability distribution on $\mathcal{S}^*$ of the white noise $\{\dot{B}(t); t\in R\}$ having the characteristic functional $C(\xi)$:

(1) $C(\xi)=\exp[-\|\xi\|^2/2], \qquad \xi\in\mathcal{S}, \ \|\cdot\|$ the $L^2(R)$-norm.

The Hilbert space (L^2) admits the Wiener-Itô decomposition

$$(L^2)=\sum_{n=0}^{\infty}\oplus\mathcal{H}_n, \tag{2}$$

where $\mathcal{H}_n$ is the *multiple Wiener integral* of degree n.

To visualize those members in (L^2) we have introduced the transformation $\mathcal{T}$ ([2]):

$$(\mathcal{T}\varphi)(\xi)=\int\exp[i\langle x,\xi\rangle]\varphi(x)d\mu(x), \qquad \varphi\in(L^2), \tag{3}$$

where $\langle\ ,\ \rangle$ stands for the canonical bilinear form that connects $\mathcal{S}$ and $\mathcal{S}^*$. The collection $\mathcal{F}=\{\mathcal{T}\varphi;\varphi\in(L^2)\}$ can be topologized so as to be