61. *Closedness of q-Ideals in a Compact and Totally Disconnected Semigroup*

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(Communicated by Kôsaku YOSIDA, M. J. A., Oct. 12, 1978)

1. A *topological semigroup* is a semigroup with a Hausdoff topology in which multiplication is continuous in both variables. In what follows S will denote a topological semigroup. An ideal P of S is termed *prime* if $AB \subset P$ implies that either $A \subset P$ or $B \subset P$, A and B being ideals of S. The notion of q-ideals has been defined in [6], namely, an ideal of S is called, briefly, a *q-ideal* if it is expressed as an intersection of open prime ideals of S.

Our main objective of this paper is to establish a necessay and sufficient condition for a q-ideal of S to be closed provided that S is compact and totally disconnected. As an application, we shall show that the radical of a compact and totally disconnected topological semigroup with zero is closed.

Throughout the whole paper we shall use the following notation.

A^* denotes the topological closure of a subset A of S.

$X-Y$ means the set of elements of X which are not in Y, where X and Y are any two sets. We write $X-y$ instead of $X-\{y\}$ when $\{y\}$ is a singleton.

E denotes the set of all idempotents in S. E is known to be a closed subset of S, and it is not empty if S is compact.

$J_0(A)$ means the union of all ideals of S which are contained in A, i.e., $J_0(A)$ is the largest ideal contained in A if $J_0(A) \neq \emptyset$, where A is a subset of S.

2. The following lemma is an analogy of the well-known result in the theory of topological groups (e.g. see [2]).

Lemma 2.1. *Let S be locally compact and totally disconnected, and let S have a right* [*or left*] *identity e. Then any neighborhood of e contains a compact subsemigroup neighborhood of e.*

Proof. Let W be any neighborhood of e. Since S is a locally compact and totally disconnected Hausdorff space, there exists a compact and open subset U of S such that $e \in U \subset W$.

Let

$$C=(S-U) \cap U^2,$$

so that C is closed. Since $Ue \cap C$ $(=U \cap C)$ is empty and U is compact,