50. *Meromorphic Functions on Compact Riemann Surfaces*

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1. By a complex space, we mean a reduced, Hausdorff, complex analytic space. Let V be a compact Riemann surface of genus g. The set $\mathrm{Hol}(V, \boldsymbol{P}^1)$ of all holomorphic maps of V into the complex projective line $\boldsymbol{P}^1$ is nothing but the set of all meromorphic functions on V. A general theorem of Douady [1] says that $\mathrm{Hol}(V, \boldsymbol{P}^1)$ is a complex space. $\mathrm{Hol}(V, \boldsymbol{P}^1)$ is divided into the open (and closed) subspaces:

$$\mathrm{Hol}(V, \boldsymbol{P}^1) = \mathrm{Const} \cup R_1(V) \cup R_2(V) \cup \cdots,$$

where Const is the set of all constant functions and $R_n(V)$ is the set of all meromorphic functions on V of (mapping) order n. Note that $R_n(V)$ is non-empty for $n \geqq g+1$. Moreover, if $n \geqq g$, then $R_n(V)$ is non-singular and of dimension $2n+1-g$ (see [3, Proposition 5]). The automorphism group $\mathrm{Aut}(\boldsymbol{P}^1)$ of $\boldsymbol{P}^1$ acts freely and properly on $R_n(V)$ (see [3]). Hence the quotient space $R_n(V)/\mathrm{Aut}(\boldsymbol{P}^1)$ is a complex space and the projection $R_n(V) \to R_n(V)/\mathrm{Aut}(\boldsymbol{P}^1)$ is a principal $\mathrm{Aut}(\boldsymbol{P}^1)$-bundle (see Holmann [2]).

It is a difficult problem to determine the integers $n \leqq g$ with non-empty $R_n(V)$ and to determine the structure of $R_n(V)$ for such n. In this note, we state the following theorems. Details will be published elsewhere.

Theorem 1. *Let $V=C$ be a non-singular plane curve of degree $d \geqq 2$. Then*

$$\mathrm{Min}\{n>0 \mid R_n(C) \textit{ is non-empty}\} = d-1.$$

If $d \geqq 3$, then $R_{d-1}(C)/\mathrm{Aut}(\boldsymbol{P}^1)$ is biholomorphic to C.

Theorem 2. *Let V be a compact Riemann surface of genus g. Let m and n be positive integers such that* (1) *m and n are relatively prime,* (2) $(m-1)(n-1) \leqq g-1$. *Then, at least one of $R_m(V)$ and $R_n(V)$ is empty.*

Corollary. *Let V be a compact Riemann surface of genus g. Let p be a prime number such that $R_p(V)$ is non-empty and let n be a positive integer such that $(p-1)(n-1) \leqq g-1$. Then,*

$$R_n(V) \begin{cases} \textit{is empty, if } n \not\equiv 0 \pmod{p} \\ \cong R_{n/p}(\boldsymbol{P}^1), \textit{ if } n \equiv 0 \pmod{p}. \end{cases}$$

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