# 39. Note on Quasi-Domination in the Sense of K. Borsuk

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In this paper we shall prove that if an approximatively 1-connected pointed continuum $(X, x_0)$ is pointed quasi-dominated by a pointed FANR, $X$ has the shape of a compact polyhedron. We shall give an example that the compactness of $Y$ in the theorem is essential. Throughout this paper all maps are continuous and by a space we mean a topological space. We mean by $\mathcal{W}$ the category of spaces having the homotopy type of $CW$-complexes and homotopy classes of maps.

Let $\underline{X}=\{X_\alpha, [p_{\alpha\alpha'}], A\}$ and $\underline{Y}=\{Y_\beta, [q_{\beta\beta'}], B\}$ be objects of pro-$\mathcal{W}$, where $[f]$ denotes the homotopy class of the map $f$. We say that $X$ is *quasi-dominated* by $\underline{Y}$ (notation: $\underline{X}\overset{q}{\leq}\underline{Y}$) if for any $\alpha_0 \in A$ there exist two system maps $\underline{f}=\{f, [f_\beta], B\}: \underline{X}\to\underline{Y}$ and $\underline{g}=\{g, [g_\alpha], A\}: \underline{Y}\to\underline{X}$ such that there exists $\alpha_1 \in A$ such that $\alpha_1 \geq \alpha_0, fg(\alpha_0)$ and $g_{\alpha_0} f_{g(\alpha_0)} p_{fg(\alpha_0)\alpha_1} \simeq p_{\alpha_0\alpha_1}$. Let $X$ and $Y$ be spaces. We say that $X$ is *quasi-dominated* by $Y$ (notation: $X\overset{q}{\leq}Y$) if there exist $\underline{X}=\{X_\alpha, [p_{\alpha\alpha'}], A\}$ and $\underline{Y}=\{Y_\beta, [q_{\beta\beta'}], B\}$ of objects of pro-$\mathcal{W}$ such that $\underline{X}$ and $\underline{Y}$ are associated with $X$ and $Y$ respectively (see [9]) and $\underline{X}\overset{q}{\leq}\underline{Y}$.

It is clear that the definition of quasi-domination of spaces is independent of chosing objects of pro-$\mathcal{W}$ associated with $X$ and $Y$. We can easily prove that for compacta our definition is equivalent to the definition of K. Borsuk [2] (cf. [7]).

Analogously the notation of pointed quasi-domination of pointed spaces (notation: $(X, x_0)\overset{q}{\leq}(Y, y_0)$) is defined.

The following is easy to prove.

**Theorem 1.** *Let $X$ and $Y$ be spaces. If $X\overset{q}{\leq}Y$ and the shape of $Y$ is trivial, the shape of $X$ is trivial.*

Thus, if a compactum $X$ is quasi-dominated by an FAR-space $Y$, then $X$ is also an FAR-space (see [1]).

From Theorem 1 the following problem is raised: *if $X$ is a compactum quasi-dominated by an* FANR-*space $Y$, is $X$ an* FANR-*space?*

We obtain the following partial answers.

**Theorem 2.** *Let $(X, x_0)$ and $(Y, y_0)$ be pointed continua. Suppose that $X$ is approximatively* 1-*connected* (*see* [1]) *and $(Y, y_0)$ is a pointed* FANR-*space. If $(X, x_0)\overset{q}{\leq}(Y, y_0)$, then $X$ has the shape of a compact*