4. *Asymptotic Properties of Solutions of n-th Order Differential Equations with Deviating Argument*

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1. Introduction. This paper is concerned with the asymptotic behavior of solutions of differential equations of the form

(A) $$Lx(t)+f(t,x(g(t)))=0,$$

where the differential operator L is defined by

$$Lx(t)=(p_{n-1}(t)(p_{n-2}(t)(\cdots(p_1(t)x'(t))'\cdots)')')'.$$

L may also be written in the form $L=L_n$, where the operators L_i are recursively defined by

$$L_1x(t)=x'(t),\quad L_ix(t)=(p_{i-1}(t)L_{i-1}x(t))',\quad 2\leqq i\leqq n.$$

The conditions we always assume for p_i, g, f are as follows:

(a) Each $p_i(t)$ is continuous and positive on $[a,\infty)$ and

$$\int_a^\infty \frac{dt}{p_i(t)}=\infty,\qquad 1\leqq i\leqq n-1;$$

(b) $g(t)$ is continuous on $[a,\infty)$ and $\lim_{t\to\infty} g(t)=\infty$;

(c) $f(t,x)$ is continuous on $[a,\infty)\times(-\infty,\infty)$ and $|f(t,x)|\leqq\omega(t,|x|)$ for $(t,x)\in[a,\infty)\times(-\infty,\infty)$, where $\omega(t,r)$ is continuous on $[a,\infty)\times[0,\infty)$ and nondecreasing in r.

Equation (A) is called *superlinear* or *sublinear* according to whether $\omega(t,r)/r$ is nondecreasing or nonincreasing in r for $r>0$.

A function $x(t)$ defined on some half-axis $[T_x,\infty)$ is said to be a solution of (A) if $L_1x(t), L_2x(t),\cdots,L_nx(t)$ exist and are continuous on (T,∞), where $T>T_x$ is such that $g(t)>T_x$ for $t>T$, and if $x(t)$ satisfies (A) on (T,∞). Hereafter our attention will be restricted to solutions of (A) which are nontrivial on any infinite subintervals of $[T_x,\infty)$. Such a solution is said to be *oscillatory* if it has arbitrarily large zeros; otherwise the solution is said to be *nonoscillatory*.

The asymptotic properties of second order functional differential equations with a general deviating argument have recently been studied by Kitamura and Kusano [2]. The object of this paper is to extend the theory developed in [2] to higher-order equations of the form (A). Of particular interest is an analysis of the effect that $g(t)$ can have on the growth or decay of solutions of equation (A) which is either superlinear or sublinear. The results are stated without proofs; an exposition in full detail will appear elsewhere.