3. *Studies on Holonomic Quantum Fields. I*

By Mikio SATO, Tetsuji MIWA, and Michio JIMBO

(Communicated by Kôsaku YOSIDA, M. J. A., April 12, 1977)

To understanding the mathematical structure of quantized fields or systems with infinite freedom, non trivial but exactly calculable models would be of great help [1]. In this and subsequent notes we present, both in the continuum and in the lattice, 2-dimensional soluble models of neutral scalar massive field theory whose τ-functions exhibit a non trivial singularity structure.

In the present article we deal with the continuum case. We introduce an auxiliary free fermi/bose field and construct the field operator by specifying its induced rotation in the space of wave functions. Making use of the "theory of rotation" (**2** cf. [2]) developed recently by the first author, we express this field operator in the normal product form of these free fields. We also calculate the asymptotic fields and the S-matrix of the field φ^F defined in 3. Next we give explicit formulae for τ-functions of these models and study their holonomy structure.

The lattice field theory will be discussed in a subsequent paper. Specifically we shall show that our model φ^F/φ_F coincide with the scaling limit of the Ising model from above/below the critical temperature. Main part of these results has been announced in [3].

We use the following notations. The space-time and the energy-momentum co-ordinates are denoted by $x=(x^0, x^1)$ and $p=(p^0, p^1)$. We also use $x^{\pm}=(x^0 \pm x^1)/2$ and $p^{\pm}=p^0 \pm p^1$. The mass-shell $\{p \in \boldsymbol{R}^2 \mid p^2=(p^0)^2-(p^1)^2=m^2\}$ $(m>0)$ is denoted by M. For $p \in M$ we set $u^{\pm 1}=p^{\pm}/m$, $\underline{du}=du/2\pi\,|u|$.

1. Let $\psi(u)^{\dagger}$ and $\psi(u)$ $(u>0)$ be the creation and annihilation operators of auxiliary fermion. If we define $\psi(u)=\psi(-u)^{\dagger}$ for $u<0$, their anti-commutation relation reads $[\psi(u), \psi(u')]_+=2\pi\,|u|\,\delta(u+u')$. Likewise we define auxiliary bosons $\phi(u)$ with the commutation relation $[\phi(u), \phi(u')]_-=2\pi u\delta(u+u')$. In two dimensional space-time these two are in fact equivalent. Namely

$$(1) \qquad \phi_{\pm}(u)=:\psi(u)\exp\int_0^{\infty}(-2)\theta(\pm(|u|-u'))\psi(u')^{\dagger}\psi(u')\underline{du'}:$$

satisfy the commutation relation $[\phi_{\pm}(u), \phi_{\pm}(u')]_-=2\pi u\delta(u+u')$, and conversely $\psi(u)$ is given by

$$(2) \qquad \psi(u)=:\phi_{\pm}(u)\exp\int_0^{\infty}(-2)\theta(\pm(|u|-u'))\phi_{\pm}(u')^{\dagger}\phi_{\pm}(u')\underline{du'}:.$$