2. *Global Solutions of the Boltzmann Equation in a Bounded Convex Domain*

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1. Introduction. We consider the Boltzmann equation

$$\frac{\partial F}{\partial t}+\sum_{i=1}^{3}\xi_i\frac{\partial F}{\partial x_i}=J(F,F), \tag{1}$$

which describes the change in time of the distribution function of the arguments space x and velocity ξ. Here $J(F,F)$ is the collision integral [1]. The equilibrium solution of (1) is $F=\omega$, where

$$\omega(\xi)=\frac{1}{(2\pi)^{3/2}}\exp\left(-\frac{1}{2}|\xi|^2\right).$$

As we are interested in solutions of (1) which are close to $F=\omega$, we introduce $f(x,\xi)$ by

$$F=\omega+\omega^{1/2}f. \tag{2}$$

Then the equation satisfied by f is

$$\frac{\partial f}{\partial t}=Bf+\Lambda\Gamma(f,f). \tag{3}$$

The explicit form of the operator B is

$$(Bf)(x,\xi)=-\sum_{i=1}^{3}\xi_i\frac{\partial f(x,\xi)}{\partial x_i}-\nu(\xi)f(x,\xi)+\int_{R^3}K(\xi,\eta)f(x,\eta)d\eta, \tag{4}$$

where $\nu(\xi)$, the collision frequency, is a certain unbounded positive function of ξ and $K(\xi,\eta)$, the collision kernel, is a symmetric function of ξ and η. The operator Λ is the multiplication operator by $\nu(\xi)$ and $\Gamma(f,f)$ denotes the quadratic term. Note that $J(\omega,\omega)=0$. We shall use Grad's estimates [1], [2] for $\nu(\xi)$, $K(\xi,\eta)$ and $\Gamma(f,f)$ in computations. This means that the potential is a hard potential in the sense of Grad and that the angular cut-off assumption is made for the differential cross section. A typical example satisfying these conditions is a gas of rigid spheres. The initial value problems for the Boltzmann equation on the torus and on the entire space have been studied earlier in [4] and [5], respectively. In this note, we treat the initial boundary value problem for the case of specular reflection boundary condition. Our

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