49. *Studies on Holonomic Quantum Fields. IV*

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This is a continuation of our previous notes [1], [2] and together with the latter constitutes the second part of the work referred to in [1]. We use the same notation as in [1], [2], [3].

1. First we shall show that the wave function $w_{F,n}={}^t(\hat{w}^1_{F,n}(x), \cdots, \hat{w}^n_{F,n}(x))$ constructed in [2] forms a basis of $W^{\mathrm{strict},R}_{a_1,\cdots,a_n}$. By (30) the local expansion of $w_{F,n}$ in the sense of (10) in [1] takes the form

$$w_{F,n}\sim\frac{i}{2}\left[\sum_{l=0}^{\infty}C_{F,l}[A]_l w-\sum_{l=0}^{\infty}\bar{C}_{F,l}w^*_l[A]\right] \tag{42}$$

where $(i/2)C_{F,l}={}^t({}^t\boldsymbol{c}_l(\hat{w}^1_{F,n}), \cdots, {}^t\boldsymbol{c}_l(\hat{w}^n_{F,n}))$. From (31) it follows that if we write $C_{F,0}=1-T$, T is purely imaginary and hermitian: $T=-\bar{T}=-{}^tT$. Since $w_{\mathfrak{R}}$ is a basis of $W^{\mathrm{strict},R}_{a_1,\cdots,a_n}$, there exists a real $n\times n$ matrix C satisfying $w_{F,n}=Cw_{\mathfrak{R}}$. Comparing the 0-th coefficients of their local expansions we have $(i/2)C_{F,0}=CC_{\mathfrak{R},0}$ or equivalently $1-T=2Ce^{-H}$. Taking the complex conjugate we have $1+T=2Ce^{H}$, and hence

$$C=(2\cosh H)^{-1},\qquad T=\tanh H=(1-G)(1+G)^{-1}. \tag{43}$$

Hence $w_{F,n}$ is also a basis of $W^{\mathrm{strict},R}_{a_1,\cdots,a_n}$.

The relation between w_F and $w_{\mathfrak{R}}$ enables us to express the coefficients B, E appearing in the system (12) in [1] satisfied by $w_{\mathfrak{R}}$, in terms of $\tau_{F,n}$ and $\tau^{\mu\nu}_{F,n}$. From (11), (40), (41) and (43) we have

$$\begin{aligned} F&=[U^{-1}V, mA], & G&=U(2\tau_{F,n}-U)^{-1}\\ B&=\sqrt{G}mA\sqrt{G}^{-1}, & E&=\sqrt{G}F\sqrt{G}^{-1}, \end{aligned} \tag{44}$$

where

$$U=\tau_{F,n}(1-T)=\begin{pmatrix} \tau_{F,n} & i\tau^{12}_{F,n} & \cdots & i\tau^{1n}_{F,n}\\ -i\tau^{12}_{F,n} & \tau_{F,n} & \cdots & i\tau^{2n}_{F,n}\\ \vdots & \vdots & \ddots & \vdots\\ -i\tau^{1n}_{F,n} & -i\tau^{2n}_{F,n} & \cdots & \tau_{F,n} \end{pmatrix}$$

$$V=2\begin{bmatrix} m^{-1}\partial_{-a_1^-}\tau_{F,n} & im^{-1}\partial_{-a_2^-}\tau^{12}_{F,n} & \cdots & im^{-1}\partial_{-a_n^-}\tau^{1n}_{F,n}\\ -im^{-1}\partial_{-a_1^-}\tau^{12}_{F,n} & m^{-1}\partial_{-a_2^-}\tau_{F,n} & \cdots & im^{-1}\partial_{-a_n^-}\tau^{2n}_{F,n}\\ \vdots & \vdots & \ddots & \vdots\\ -im^{-1}\partial_{-a_1^-}\tau^{1n}_{F,n} & -im^{-1}\partial_{-a_2^-}\tau^{2n}_{F,n} & \cdots & m^{-1}\partial_{-a_n^-}\tau_{F,n} \end{bmatrix}. \tag{45}$$

Thus we have constructed, in terms of ψ, φ_F and φ^F, not only a solution to the extended holonomic system (12) but also one to the system of total differtntial equations (18).