47. *Periods of Primitive Forms*

By Kazuyuki HATADA
Department of Mathematics, Faculty of Science, University of Tokyo

(Communicated by Kunihiko KODAIRA, M. J. A., Oct. 12, 1977)

Introduction. We combine Shapiro's lemma on cohomology of groups with Eichler-Shimura isomorphism for elliptic modular forms. As an application of it, we show the rationality of the periods of any primitive cusp form of Neben type. Details will appear elsewhere.

§1. Let Γ be a congruence subgroup of $SL(2, \boldsymbol{Z})$. Γ acts on the complex upper half place H from the left by $\begin{pmatrix} a & b \\ c & d \end{pmatrix}(z)=(az+b)/(cz+d)$ for $z \in H$. Let $S_{w+2}(\Gamma)$ be the space of cusp forms of weight $w+2 \geqq 2$ on Γ, and $S^{\boldsymbol{R}}_{w+2}(\Gamma)$ be the subspace of $S_{w+2}(\Gamma)$ consisting of the cusp forms whose Fourier coefficients at $z=i\infty$ are all real. Let P be the set of all the parabolic elements in $SL(2, \boldsymbol{Z})=\Gamma(1)$. Let $d\vec{z}_w$ be the $(w+1)$ dimensional differential form, the transpose of $(dz, zdz, z^2dz, \cdots, z^w dz)$ on the H. Let ρ_w be the representation of Γ; $\Gamma \to GL(w+1, \boldsymbol{Z})$, which is given by $(cz+d)^{w+2}(d\vec{z}_w \circ g)=\rho_w(g)(d\vec{z}_w)$ for all $g=\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma$, where $(d\vec{z}_w) \circ g$ denotes the pull back of $d\vec{z}_w$ by g. Let $\eta_w = \operatorname*{Ind}_{\Gamma \uparrow \Gamma(1)} \rho_w$ be the representation of $\Gamma(1)$ induced from ρ_w. Let $H^1_{P \cap \Gamma}(\Gamma, \rho_w, R)$ and $H^1_P(\Gamma(1), \eta_w, R)$ be the first parabolic cohomology group with R coefficients where $R=\boldsymbol{R}$ or $\boldsymbol{Z}$. $\boldsymbol{R}$, $\boldsymbol{Q}$ and $\boldsymbol{Z}$ denote the real numbers, the rational numbers and the rational integers respectively. Let $g_1=1, g_2, g_3, \cdots, g_m$ be representative of the left coset decomposition $\Gamma \backslash \Gamma(1)$. For a $f \in S_{w+2}(\Gamma)$, we set $\mathscr{D}(f)=$ the $(w+1)m$ dimensional differential form which is given by $\begin{pmatrix} (f(z)d\vec{z}_w) \circ g_1 \\ (f(z)d\vec{z}_w) \circ g_2 \\ \vdots \\ (f(z)d\vec{z}_w) \circ g_m \end{pmatrix}$, where $(f(z)d\vec{z}_w) \circ g$ denotes the pull back of $(f(z)d\vec{z}_w)$ by $g \in \Gamma(1)$. We normalize η_w such as $\eta_w(g)\mathscr{D}(f) = \mathscr{D}(f) \circ g$. Now let z_0 be any point in the H, $\vec{A}$ be any $(w+1)m$ dimensional column vector in $\boldsymbol{R}^{(w+1)m}$ and w be an arbitrary rational integer $\geqq 0$. Then we have:

Lemma 1. For a $f \in S_{w+2}(\Gamma)$, $\Gamma(1) \ni \sigma \mapsto \operatorname{Re} \int_{z_0}^{\sigma z_0} \mathscr{D}(f) + (\eta_w(\sigma)-1)\vec{A}$ is a cocycle in $Z^1_P(\Gamma(1), \eta_w, \boldsymbol{R})$. Its cohomology class in $H^1_P(\Gamma(1), \eta_w, \boldsymbol{R})$ is determined by f and independent of z_0 and $\vec{A}$.

Theorem 1. *There is an* $\boldsymbol{R}$*-linear surjective isomorphism*