30. *On the Mixed Problem with d'Alembertian in a Quarter Space*

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Introduction. In this note we consider the mixed problem

$$(0.1)\quad \begin{cases} \Box u \equiv (D_t^2 - D_x^2 - \sum_{j=1}^{n-1} D_{y_j}^2)u = f(t,x,y) & \text{in } (0,\infty)\times \boldsymbol{R}_+^n, \\ Bu \equiv (D_x + b_0(t,y)D_t + \sum_{j=1}^{n-1} b_j(t,y)D_{y_j} + c(t,y))u|_{x=0} & \\ \quad = g(t,y) & \text{on } (0,\infty)\times \boldsymbol{R}^{n-1}, \\ D_t u|_{t=0} = u_1(x,y) & \text{on } R_+^n, \\ u|_{t=0} = u_0(x,y) & \text{on } \boldsymbol{R}_+^n, \end{cases}$$

where $D_t = -i\partial/\partial t, D_x = -i\partial/\partial x, \cdots, c(t,y) \in \mathcal{B}^\infty(\bar{\boldsymbol{R}}_+^1 \times \boldsymbol{R}^{n-1})$[1] and $b_j(t,y)$ $(j=0,1,\cdots,n-1)$ are real-valued functions belonging to $\mathcal{B}^\infty(\bar{\boldsymbol{R}}_+^1\times \boldsymbol{R}^{n-1})$. Let us say that (0.1) is C^∞ well-posed when there exists a unique solution $u(t,x,y)$ in $C^\infty(\bar{\boldsymbol{R}}_+^1 \times \bar{\boldsymbol{R}}_+^n)$ for any $(u_0, u_1, f, g) \in C^\infty(\bar{\boldsymbol{R}}_+^n)\times C^\infty(\bar{\boldsymbol{R}}_+^n) \times C^\infty(\bar{\boldsymbol{R}}_+^1\times \bar{\boldsymbol{R}}_+^n)\times C^\infty(\bar{\boldsymbol{R}}_+^1 \times \boldsymbol{R}^{n-1})$ satisfying the compatibility condition of infinite order.

When $b_0, \cdots, b_{n-1}$ and c are all constant, by Sakamoto [4] we know a necessary and sufficient condition for C^∞ well-posedness. If $b_0<1$ (0.1) is C^∞ well-posed, and in the case $n \geqq 3$ it is so only if $b_0<1$. Agemi and Shirota in [1] studied (0.1) precisely when $n=2$, $c=0$ (b_j is constant). Tsuji in [6] treated the case that $b_0, \cdots, b_{n-1}$ and c are variable, and showed the existence of the solution in the Sobolev space. Furthermore, he stated that the Lopatinski condition must be satified at any point if (0.1) is C^∞ well-posed. Ikawa [2] investigated (0.1) in a general domain in the case $n=2$, $b_0=0$.

In our note we shall study C^∞ well-posedness and the propagation speed of (0.1). Consider the following equation in λ:

$$\sqrt{1-\lambda^2} = b_0(t,y) + |b'(t,y)|\lambda \qquad (b' = (b_1, \cdots, b_{n-1})).$$

Then, if $b_0(t,y)<1$ this equation has a positive root or no real root. In the former case we denote the positive root by $\lambda_0(t,y)$, and in the latter case set $\lambda_0(t,y)=1$.

Theorem 1. *If* $\sup_{(t,y)\in \boldsymbol{R}_+^1\times \boldsymbol{R}^{n-1}} b_0(t,y) < 1$, *then* (0.1) *is* C^∞ *well-posed and has a finite propagation speed less than* $\sup_{(t,y)\in \boldsymbol{R}_+^1 \times \boldsymbol{R}^{n-1}} \lambda_0(t,y)^{-1}$.

For a constant $v>0$ we set $C_v(t_0, x_0, y_0) = \{(t,x,y) : (t-t_0)v + ((x-x_0)^2$

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1) $\mathcal{B}^\infty(M)$ denotes the set $\{h(z) \in C^\infty(M) ; |h|_m = \sum_{|\alpha|\leqq m} |D_z^\alpha h(z)| < \infty$ for $m=0,1,\cdots\}$.