85. *Probability theoretic Investigations on Inheritance. II₁. Cross-Breeding Phenomena.*

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1. Limit distribution of composed population.

In a population, the distribution of an inherited character will arrive at an equilibrium state soon in the next generation, if matings take place at random with respect to the character, even when the distribution in initial generation is not in an equilibrium state. This is a fact that we have already noticed at the end of the previous paper[1]. In practice, however, particularly in such a case where the population cover a very wide range, it will not be expected that such random matings take place for once only. We should rather expect that this buffer effect grows gradually through many generations. Although we shall postpone the detailed discussion of this problem still later, we notice here a remarkable fact in a cross-breeding process.

As in the last section of I, we consider g races $X^{(\nu)}$ each of which possesses an equilibrium distribution. Denoting by

$$(1.1)\qquad p_i^{(\nu)} \qquad (i = 1, \dots, m)$$

the frequencies of genes A_i in the race $X^{(\nu)}$, then the frequencies of genotypes are given by

$$(1.2)\qquad \bar{A}_{ii}^{(\nu)} = p_i^{(\nu)2}, \qquad \bar{A}_{ij}^{(\nu)} = 2p_i^{(\nu)}p_j^{(\nu)} \quad (i \neq j).$$

These quantities satisfy of course the fundamental relations

$$\sum_{i=1}^{m} p_i^{(\nu)} = 1, \quad \sum_{i \leq j} \bar{A}_{ij}^{(\nu)} = 1 \qquad (\nu = 1, \dots, g)$$

and moreover, as shown in (2.9) of I, we have

$$(1.3)\quad p_i^{(\nu)} = \bar{A}_{ii}^{(\nu)} + \frac{1}{2}\sum_{j \neq i} A_{ij}^{(\nu)} \quad (i = 1, \dots, m\,;\ \nu = 1, \dots, g).$$

Suppose now, as in § 4 of I, that these races $X^{(\nu)}$ $(\nu = 1, \dots, g)$ are mixed at the rate $\lambda^{(\nu)} (> 0)$ with

$$(1.4)\qquad \sum_{\nu=1}^{g} \lambda^{(\nu)} = 1$$

the distribution of resulting population X will then be given by

$$(1.5)\qquad \bar{A}_{ii} = \sum_{\nu=1}^{g} \lambda^{(\nu)} \bar{A}_{ii}^{(\nu)}, \quad \bar{A}_{ij} = \sum_{\nu=1}^{g} \lambda^{(\nu)} \bar{A}_{ij}^{(\nu)}.$$

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1) Y. Komatu, Probability-theoretic investigations on inheritance. I. Distribution of genes. Proc. Jap. Acad., **27** (1951), 371–377. This will be referred to as I.