84. *Probability-theoretic Investigations on Inheritance. I. Distribution of Genes.*

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0. Introduction.

The inheritance phenomenon has found increasing application in various branches of biology and medicine. Its theoretical foundation, especially from probability-theoretic or statistical point of view, has together been attempted. Most of such studies has treated, however, the problems only separately corresponding to individual concrete cases. It is the purpose of this paper and its subsequent papers to investigate the phenomenon based upon a general mode of inheritance from probability-theoretic standpoint in detail and to develop a unified theory in a systematic manner.

1. Mode of inheritance to be discussed.

Let us consider a single inherited character. The character of every individual is composed of a pair of genes each of which has originated from each of its parents through their sexual cells. We suppose now, in general, that the character in question consists of m distinct genes denoted by A_i $(i = 1, \ldots, m)$, the inheritance of which is subject to Medelian law. The possible genotypes may then be denoted by A_iA_j $(i, j = 1, \ldots, m)$.

But, since the order of genes in a genotype is immaterial, both genotypes A_iA_j and A_jA_i for different suffices i, j must also be regarded as identical each other. Hence, introducing the abbreviated notation

$$A_{ij} = A_iA_j ,$$

the symmetry relations $A_{ij} = A_{ji}$ for any i and j follow immediately. In view of these relations, the number of different genotypes amounts to

$$\frac{1}{2}m(m+1) ; \tag{1.1}$$

there exist m homozygotes A_{ii} $(i = 1, \ldots, m)$ and $\frac{1}{2}m(m-1)$ heterozygotes A_{ij} $(i, j = 1, \ldots, m ; i < j)$.

If a gene A_j is recessive against A_i, then the genotypes A_{ii} and A_{ij} present both, as a phenotype, the character A_i, while the genotype A_{jj} alone presents the character A_j. This dominance