90. On the Adiabatic Theorem for the Hamiltonian System of Differential Equations in the Classical Mechanics. III

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6. We consider the Hamiltonian system containing a parameter $\lambda(>0)$

(14) $$dp/dt=-\lambda\partial H/\partial q(p,q,t),\ dq/dt=\lambda\partial H/\partial p(p,q,t)$$

in D. If $(p^0,q^0,t^0)\in D$ and $\lambda>0$, there is a unique solution of (14) in D passing through $(p^0,q^0,t^0,)$ and prolonged as far as possible to the both directions of the time t, by the regularity of $H(p,q,s)$ in Assumption 1.[1] We denote it by

(15) $$p=\tilde{\mathfrak{p}}(t,p^0,q^0,t^0,\lambda),\ q=\tilde{\mathfrak{q}}(t,p^0,q^0,t^0,\lambda).$$

For a fixed $(p^0,q^0,t^0)\in D$ and a fix $\lambda(>0)$, $\tilde{\mathfrak{p}}(t,p^0,q^0,t^0,\lambda)$, $\tilde{\mathfrak{q}}(t,p^0,q^0,t^0,\lambda)$ are defined on a subinterval of the time interval $a\leqq t\leqq b$ which may be open, closed or half-open according to (p^0,q^0,t^0,λ).[1]

Since $\partial\tilde{\mathfrak{J}}/\partial s$ is continuous on $\bar{D}$ and $\bar{D}$ is compact, there is a number $M(>0)$ such that

(16) $$|\partial\tilde{\mathfrak{J}}/\partial s|\leqq M \qquad \text{on } D.$$

THEOREM 3. *Let a' and b' be two numbers such that $a\leqq a'<b'\leqq b$ and $(b'-a')<(J_2^*-J_1^*)/(2M)$ and let us put $J_2=J_2^*-M(b'-a')$, $J_1=J_1^*+M(b'-a')$. Then the solution of* (14) *passing through (p^0,q^0,a') where $(p^0,q^0)\in I(J_1,J_2,a')$ can be prolonged in D to the time interval $a'\leqq t\leqq b'$ for every $\lambda(>0)$.*

PROOF. Let β be the least upper bound of β' such that the solution in D of (14), $p=\tilde{\mathfrak{p}}(t,p^0,q^0,a',\lambda)$, $q=\tilde{\mathfrak{q}}(t,p^0,q^0,a',\lambda)$ for a fixed $(p^0,q^0)\in I(J_1,J_2,a')$ and a fixed $\lambda>0$, can be defined for the time interval $a'\leqq t<\beta'$ and such that $a'<\beta'\leqq b'$. Then $a'<\beta\leqq b'$ and this solution in D can be defined on the the time interval $a'\leqq t<\beta$. Since $\partial H/\partial p, \partial H/\partial q$ are bounded on D by their continuity on the compact set $\bar{D}$, the functions $\tilde{\mathfrak{p}}_i(t,p^0,q^0,a',\lambda)$ $\tilde{\mathfrak{q}}_i(t,p^0,q^0,a',\lambda)$ $(i=1,\cdots,n)$ of t representing a solution of (14) in D, are uniformly continuous on the interval $a'\leqq t<\beta$. Hence the limits

$$\tilde{\mathfrak{p}}(t,p^0,q^0,a',\lambda)\to p'(t\to\beta-0)$$
$$\tilde{\mathfrak{q}}(t,p^0,q^0,a',\lambda)\to q'(t\to\beta-0)$$

exist and $(p',q',\beta)\in\bar{D}$.

We shall sometimes abbreviate $\tilde{\mathfrak{p}}(t,p^0,q^0,a',\lambda)$ and $\tilde{\mathfrak{q}}(t,p^0,q^0,a',\lambda)$

1) Cf. E. Kamke [1, pp. 135–136 and pp. 137–142].