89. On the Adiabatic Theorem for the Hamiltonian System of Differential Equations in the Classical Mechanics. II

By Takashi KASUGA

Department of Mathematics, University of Osaka

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3. Let (X, m) be a measure space where m is a finite, separable, and complete measure[1] defined on a Borel field in X. A one-parameter group $\{\mathfrak{T}_t \mid -\infty < t < +\infty\}$ of one-to-one mappings $\mathfrak{T}_t$ of X onto X is called a flow on (X, m). A measurable function $f(P)$ on (X, m) is called an invariant function of a flow $\{\mathfrak{T}_t\}$ on (X, m) if

$$f(\mathfrak{T}_t(P)) = f(P)$$

almost everywhere on (X, m) for every fixed t and it is called a strictly invariant function of a flow $\{\mathfrak{T}_t\}$ on (X, m) if it is defined everywhere on X and

$$f(\mathfrak{T}_t(P)) = f(P)$$

for all (P, t) such that $P \in X, -\infty < t < +\infty$. A measure-preserving and measurable flow[2] $\{\mathfrak{T}_t\}$ on (X, m) is ergodic (in the sense of J. v. Neumann) if and only if all its invariant functions are equivalent[3] to constants on (X, m). If a flow $\{\mathfrak{T}_t\}$ on (X, m) is measure-preserving and measurable, then we can associate with it a one-parameter group $\{\mathfrak{U}_t \mid -\infty < t < +\infty\}$ of unitary transformations $\mathfrak{U}_t$ on $L^2(X, m)$ by

$$(\mathfrak{U}_t f)(P) = f(\mathfrak{T}_t(P)) \quad f \in L^2(X, m), \quad P \in X$$

and $\mathfrak{U}_t$ is continuous as a function of t in the strong topology of $\mathfrak{U}_t$.[4]

If X is a Lebesgue measurable subset of a Euclidean space R^r and m is the usual Lebesgue measure in R^r defined for all Lebesgue measurable subsets of X, a flow on (X, m) is simply called a flow on X in the following and we write simply $L^2(X)$ for $L^2(X, m)$.

4. We consider the Hamiltonian system with a parameter s

$$(9) \qquad dp/dt = -\partial H/\partial q(p, q, s) \quad dq/dt = \partial H/\partial p(p, q, s).$$

By Assumption 1, the solution of (9)

$$(10) \qquad p = \mathfrak{p}(t, p^0, q^0, s) \quad q = \mathfrak{q}(t, p^0, q^0, s)$$

in the open set $I(s)$ for a fixed s $(a \leqq s \leqq b)$ with the initial conditions $(p, q) = (p^0, q^0)((p^0, q^0) \in I(s))$ at $t=0$, can be uniquely prolonged for the

1) For the definition of complete or separable measure, cf. P. Halmos [1].

2) For the definition of a measure-preserving, a measurable or an ergodic flow on (X, m), cf. E. Hopf [2, pp. 8–9 and p. 28].

3) Two measurable functions on (X, m) are called equivalent on (X, m) if they coincide almost everywhere on (X, m).

4) For definitions and results concerning flows on a measure space used in this paper, cf. E. Hopf [2].