84. *Harmonic Condition in the Theory of Unified Field*

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(Comm. by K. KUNUGI, M.J.A., July 12, 1961)

ABSTRACT. The linearised equations of a modified version of Einstein's field theory have their aspect deeply changed if one assumes that the cosmological constant, called here fundamental, may take very high values with respect to the antisymmetrical field of the electromagnetism.

One justifies then, physically, the choice of a generalised condition of harmonicity which leads through the gravitational equations of the theory to the definition of a mass to charge ratio.

The application of a variational principle on the fields $L^{\alpha}_{\beta\gamma}$, $\mathfrak{G}^{\mu\nu}$, S_μ, a, $\mathfrak{A}^\mu$ through the following modified*) lagrangian:

(1) $$\mathfrak{L}\equiv\mathfrak{G}^{\mu\nu}[W_{\mu\nu}-m\Pi^{-}_{\underset{\vee}{\mu\nu}}]+2\mathfrak{A}^\mu L_\mu+a(\mathfrak{G}^{\mu\nu}S_\mu S_\nu-2\sqrt{-g}\,\alpha^2)$$

leads if one assumes that the antisymmetric part of the fundamental tensor $\varphi_{\mu\nu}$ is small enough $\simeq\varepsilon$ and after that the equations in series of powers of ε have been expanded, to the approximate equations [4]:

(2) $$\Box\varphi_{\mu\nu}=K_0\Pi^{-}_{\mu\nu}-G^{\tau\rho}_{\cdot\mu\nu}\varphi_{\tau\rho}+G_{\tau\nu}\varphi^{\tau}_{\cdot\mu}-G_{\tau\mu}\varphi^{\tau\nu}_{\cdot\cdot}+2a\alpha^2\varphi_{\mu\nu}+2KS^{\lambda}_{\cdot}\nabla_\lambda\varphi_{\mu\nu}$$
$$+\frac{K}{6}(\nabla_\mu\varphi_{\nu\tau}-\nabla_{\cdot}\varphi_{\mu\tau})S^{\tau}_{\cdot}-\frac{2K}{3}[\nabla_\nu(\varphi_{\sigma\mu}S^{\sigma}_{\cdot})-\nabla_\mu(\varphi_{\sigma\nu}S^{\sigma})]$$

(3) $$G_{\mu\nu}+aS_\mu S_\nu-a\alpha^2\gamma_{\mu\nu}=-\frac{1}{6}[\nabla_\mu(KS_\nu)+\nabla_\nu(KS_\mu)]+\frac{K^2}{2}S_\mu S_\nu$$
$$+\frac{1}{2}\nabla^{\rho}_{\cdot}(\varphi^{\sigma}_{\cdot\nu}(\nabla_\mu\varphi_{\rho\sigma}-\nabla_\sigma\varphi_{\mu\rho}+\nabla_\rho\varphi_{\sigma\mu})+\text{sym. for }\mu,\nu]$$
$$-\frac{1}{6}\nabla^{\rho}_{\cdot}[KS^{\lambda}_{\cdot}(\varphi_{\mu\lambda}\gamma_{\nu\rho}+\varphi_{\nu\lambda}\gamma_{\mu\rho}-3\gamma_{\mu\nu}\varphi_{\rho\lambda})]$$
$$-\Big(-\frac{1}{2}\varphi_{\mu\rho\cdot}^{\ \ \ \lambda}+\nabla^{\lambda}_{\cdot}\varphi_{\mu\rho}\Big)\Big(-\frac{1}{2}\varphi_{\lambda\nu\cdot}^{\ \ \ \rho}+\nabla^{\rho}_{\cdot}\varphi_{\lambda\nu}\Big)$$

(where $\Box$ represents $\nabla^{\lambda}_{\cdot}\nabla_\lambda$, ∇ the covariant derivative in the γ metric: $\gamma_{\mu\nu}\equiv g_{\underline{\mu\nu}}$ symmetric, the dotted indices being raised or lowered by γ $K_0=a/3m+2m$; $K=a/m$; $G_{\mu\nu}=G^{\rho}_{\cdot\mu\nu\rho}$ (Ricci))

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*) This change has been brought [1] in order to elude particular limits of the theory and to give the opportunity to arrive at $\partial_\lambda\mathfrak{G}^{\underset{\vee}{\mu\tau}}\neq 0$; $L^{\alpha}_{\beta\gamma}$ is the affine connexion of the Cartan's generalised space, to which we assign the condition $L_\alpha\equiv L^{\rho}_{\underset{\vee}{\alpha\rho}}=\frac{1}{2}(L^{\rho}_{\alpha\rho}-L^{\rho}_{\rho\alpha})$ through the multiplicator $\mathfrak{A}^\mu\cdot\mathfrak{G}^{\mu\nu}=\sqrt{-g}g^{\mu\nu}$ fundamental density, S_μ phenomenological vector holding the normalisation condition in $2\alpha^2$ assigned by the multiplicator a. We will assume in what follows, in order to get a formal simplification, a practically constant.