47. *Algebraic Equaton, whose Roots lie in a Unit Circle or in a Half-plane.*

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I. *Algebraic equations, whose roots lie in a unit circle.*

1. In this paper $\bar{a}$ means the conjugate complex of a. Let

$$f(x)=a_0+a_1x+\dots+a_nx^n,\quad f^*(x)=x^n\bar{f}\Big(\frac{1}{x}\Big)=\bar{a}_n+\bar{a}_{n-1}x+\dots+\bar{a}_0x^n, \tag{1}$$

$$A=\begin{pmatrix} a_0, & a_1, & a_2, & \dots, & a_{n-1} \\ 0, & a_0, & a_1, & \dots, & a_{n-2} \\ 0, & 0, & a_0, & \dots, & a_{n-3} \\ \dots & & & & \\ 0, & 0, & 0, & \dots, & a_0 \end{pmatrix},\qquad \bar{A}'=\begin{pmatrix} \bar{a}_0, & 0, & 0, & \dots, & 0 \\ \bar{a}_1, & \bar{a}, & 0, & \dots, & 0 \\ \bar{a}_2, & \bar{a}_1, & \bar{a}_0, & \dots, & 0 \\ \dots & & & & \\ \bar{a}_{n-1}, & \bar{a}_{n-2}, & \bar{a}_{n-3}, & \dots, & \bar{a}_0 \end{pmatrix},$$

$$B=\begin{pmatrix} \bar{a}_n, & \bar{a}_{n-1}, & \bar{a}_{n-2}, & \dots, & \bar{a}_1 \\ 0, & \bar{a}_n, & \bar{a}_{n-1}, & \dots, & \bar{a}_2 \\ 0, & 0, & \bar{a}_n, & \dots, & \bar{a}_3 \\ \dots & & & & \\ 0, & 0, & 0, & \dots, & \bar{a}_n \end{pmatrix},\qquad \bar{B}'=\begin{pmatrix} a_n, & 0, & 0, & \dots, & 0 \\ a_{n-1}, & a_n, & 0, & \dots, & 0 \\ a_{n-1}, & a_{n-1}, & a_n, & \dots, & 0 \\ \dots & & & & \\ a_1, & a_2, & a_3, & \dots, & a \end{pmatrix},$$

$$\mathfrak{H}=\bar{B}'B-\bar{A}'A=(\gamma_{ik}),\ |\mathfrak{H}|=\text{det.}\ (\gamma_{ik}),$$

$$\mathfrak{H}(x)=\sum_0^{n-1}\gamma_{ik}x_i\,\bar{x}_k,\ (\gamma_{ki}=\bar{\gamma}_{ik}), \tag{2}$$

$$\delta_\nu=\left|\begin{array}{cccc|cccc} a_n, & 0, & \dots, & 0, & a_0, & a_1, & \dots, & a_{\nu-1} \\ a_{n-1}, & a_n, & \dots, & 0, & 0, & a_0, & \dots, & a_{\nu-2} \\ \dots & & & & \dots & & & \\ a_{n-\nu+1}, & a_{n-\nu+2}, & \dots, & a_n, & 0, & 0, & \dots, & a_0 \\ \hline \bar{a}_0, & 0, & \dots, & 0, & \bar{a}_n, & \bar{a}_{n-1}, & \dots, & \bar{a}_{n-\nu+1} \\ \bar{a}_1, & \bar{a}_0, & \dots, & 0, & 0, & \bar{a}_n, & \dots, & \bar{a}_{n-\nu+2} \\ & & & & \dots & & & \\ \bar{a}_{\nu-1}, & \bar{a}_{\nu-2}, & \dots, & \bar{a}_0, & 0, & 0, & \dots, & \bar{a}_n \end{array}\right|,\ (\nu=1, 2,\dots, n). \tag{3}$$

We denote the determinant of a matrix A by $|A|$ and its ν-th section by A_ν, which is a matrix formed with elements of A lying in the first ν rows and