43. Note on the theory of conformal representation by meromorphic functions II.*)

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§ 4. *Limitations with regard to pole.*

We turn next our attention to the problem relating to the pole z_∞ and its residue A, supposing that there exists at all a pole of the function $f(z)$, schlicht and meromorphic in the basic circle $|z|<1$. We consider now, for that purpose, again as in the part I the corresponding function $g(\zeta)=f(\zeta^{-1})^{-1}$ and make use of a distortion formula

$$(4.1)\qquad |\lg g'(\zeta)| \leqq \lg \frac{|\zeta|^2}{|\zeta|^2-1} \qquad (|\zeta|>1)$$

discovered first by Grunsky[15] and given later otherwise by Golusin[16] which sharpens a classical theorem

$$(4.2)\qquad |g'(\zeta)| \leqq \frac{|\zeta|^2}{|\zeta|^2-1} \qquad (|\zeta|>1)$$

due to Löwner.[17] The logarithmic function in the left-hand side in (4.1) means, of course, such a branch that vanishes at $\zeta=\infty$. The formula (4.1) is, in reality, more profound than all the others used in this paper, save the previously quoted one due to Landau.

We state now the following proposition.

Theorem. *If $f(z)$ is schlicht in $|z|<1$ and possesses there actually a pole z_∞ with residue A, then we have*

$$(4.3)\qquad \left|\lg \frac{-z_\infty^2}{A}\right| \leqq \lg \frac{1}{1-|z_\infty|^2},$$

and hence especially

$$(4.4)\qquad |z_\infty|^2(1-|z_\infty|^2) \leqq |A| \leqq \frac{|z_\infty|^2}{1-|z_\infty|^2}$$

and

$$(4.5)\quad \arg(-z_\infty^2)+\lg(1-|z_\infty|^2) \leqq \arg A \leqq \arg(-z_\infty^2)-\lg(1-|z_\infty|^2),$$

*) I. Proc. **21** (1945), 269.

15) H. Grunsky, Neue Abschätzungen zur konformen Abbildung ein- und mehrfach zusammenhängender Bereiche. Schriften d. math. Sem. u. d. Inst. f. angew. Math. d. Univ. Berlin **1** (1932/3), 95–140.

16) G. M. Golusin, Ergänzung zur Arbeit „Über die Verzerrungssätze der schlichten konformen Abbildungen". Recueil Math. **2** (**44**) (1938), 685–688 (in Russian).

17) K. Löwner, loc. cit. 9). Cf. also E. Frank, Beiträge zur konformen Abbildung. Inaug.-Diss. Frankfurt (1919).