42. *Note on the theory of conformal representation by meromorphic functions. I.*

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§ 1. *Preliminaries.*

We consider, in general, on the one hand a family of analytic functions

$$(1.1) \qquad \{g(\zeta)\},\quad g(\infty)=\infty,\ g'(\infty)=1,$$

defined on $|\zeta|>1$ and normalized at $\zeta=\infty$, as is here explicitly written, and on the other a family of analytic functions

$$(1.2) \qquad \{f(z)\},\quad f(0)=0,\qquad f'(0)=1,$$

defined in $|z|<1$ and normalized at $z=0$. We can then establish a ono-to-one correspondence between them by the relations

$$(1.3) \quad \zeta z=1,\quad g(\zeta)f(z)=1 \quad \text{i.e.}\quad g(\zeta)=f(\zeta^{-1})^{-1},\ f(z)=g(z^{-1})^{-1};$$

the corresponding functions $g(\zeta)$ and $f(z)$ behave, furtheremore, both analytic and *schlicht* (univalent) at the same time in their respective domains of definition, the case to which we shall confine ourselves in the following lines. Under these circumstances any properties of the one family imply at once the corresponding ones of the other. As a matter of fact, it is especially remarkable that the so-called Bieberbach's area theorem concerning the former family has paved a way also to a systematic development of the theory of the latter.

But the considerations on the latter are often confined to the sub-family, consisting of regular functions only, that is to say, consisting of only those functions $f(z)$ which correspond, by (1.3), to special functions $g(\zeta)$ vanishing nowhere on $|\zeta|>1$. Various properties of this sub-family have been hitherto, indeed partly by an essential utilization of the supplementary restriction in question, i.e. the regularity, investigated in full detail. When the family (1.1) is, however, supposed to be merely schlicht, we should rather consider the schlicht and generally *meromorphic* family (1.2) itself which correspond, by the relations (1.3), exactly to the whole family (1.1). The results on the family just ranged have been, though often of importance and very useful, established hitherto comparatively few.

Even if we assume that the family (1.2) of schlicht functions are meromorphic, each member $f(z)$ has, as a matter of course, at most only one pole of the first order in the basic circle $|z|<1$. We shall however consider here, from the