

30. Conics in D van Dantzig's projective space.

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§ 0. In his very interesting papers, H. Hombu¹⁾ has developed the projective theory of a system of paths of higher order. He has also treated, as an application of his general theory, the system of paths of the third order defined by the differential equations of the form

$$(0.1) \quad T^i \equiv x^{(3)i} + A_k^i x^{(2)k} + B^i = 0^{(3)}, \quad (i, j, k, \dots = 1, 2, \dots, n)$$

where $A_k^i(x, x^{(1)})$ and $B^i(x, x^{(1)})$ are homogeneous functions of the degree $+1$ and $+3$ respectively in $x^{(1)i}$, $x^{(r)i}$ denoting the ordinary r -th derivative with respect to the parameter chosen.

On the other hand, M. Mikami²⁾ has studied the parabolas in the so-called generalized spaces, say, in the spaces of line-elements $(x, x^{(1)})$ of the first order with an affine connection $\Gamma_{jk}^i(x, x^{(1)})$, the parallel displacement of a vector v^i being defined by the vanishing of the covariant differential

$$(0.2) \quad \delta v^i = dv^i + \Gamma_{jk}^i v^j dx^k.$$

M. Mikami has defined parabolas by the differential equations of the form

$$(0.3) \quad \frac{\delta^2}{ds^2} x^{(1)i} = 0,$$

as a natural generalization of parabolas in an ordinary affine space. If we write down fully the equations (0.3), we obtain the equations of the form (0.1). Then, what is the necessary and sufficient condition that the system of paths (0.1) defines a system of parabolas? The answer to this question was also given by M. Mikami.

H. Hombu and M. Mikami³⁾ have continued this study of parabolas in the generalized spaces of paths of J. Douglas. They have considered the contacts of

1) H. Hombu: Die projektive Theorie eines Systems der "paths" höherer Ordnung, I, Japanese Journal of Math., **15** (1938), 139-196; II, Journ. Fac. Sc. Hokkaido Imp. Univ., (I) **7** (1938), 35-94.

2) H. Hombu: Die projektive Theorie der "paths" $x^{(3)i} + A_k^i x^{(2)k} + B^i = 0$. Proc., **13** (1937), 410-413.

3) M. Mikami: On parabolas in the generalized spaces. Japanese Journal of Math., **17** (1940), 185-200.

4) H. Hombu and M. Mikami: Parabolas and projective transformations in the generalized spaces of paths. Japanese Journal of Math., **17** (1941), 307-335.