12. *An Evaluation in the Theory of Multivalent Functions.*

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(Comm. by S. KAKEYA, M. I. A., Apr. 12, 1946.)

1. In a previous paper we have considered a family $\mathfrak{F}_r$ of analytic functions which are regular and p-valent in the unit-circle $|z|<1$ and have the expansion of the type

(1) $$w(z) = z^p + a_{p+1} z^{p+1} + \dots\dots ;$$

and proved that

$$|w(z)| \geqq \left(\frac{1}{1.0365\dots}\right)^p \cdot \frac{|z|^p}{(1+|z|)^{2p}}$$

for $|z| \leqq x_0$ and

$$|w(z)| \geqq \left(\frac{1}{1.0604\dots}\right)^p \cdot \frac{|z|^p}{(1+|z|)^{2p}}$$

for $x_0 \leqq |z| \leqq 1$, where $x_0 = 0.7389\dots\dots$

2. Here we want to ameliorate this result, and our new evaluation is as follows:

For $|z| \leqq x_1$

(2) $$|w(z)| \geqq \left(\frac{1}{1.00755\dots}\right)^p \cdot \frac{|z|^p}{(1+|z|)^{2p}}$$

and for $x_1 \leqq |z| \leqq 1$

(3) $$|w(z)| \geqq \left(\frac{1}{1.03142\dots}\right)^p \cdot \frac{|z|}{(1+|z|)^{2p}},$$

where $x_1 = 0.80458\dots$

We will give here an outline of the demonstration of this result, and the detailed proof shall be given in another journal.

3. Our evaluation is based on the following theorems:

(I) $|a_{p+1}| \leqq 2p$[2]

(II) $$|w(z)|^{\frac{3}{2p}} \geqq \frac{|z|^{\frac{3}{2}}}{1+3|z|+2\sqrt{3}\,|z|\sqrt{\frac{|z|}{2}\log\frac{1+|z|}{1-|z|}}}, \quad \text{for } |z|<1.$$

The sketch of the proof of the inequality (II) shall be given in the following lines.

If we write

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1) A. Kobori, Zur Theorie der mehrwertigen Funktionen. Japanese Journ. of Math. Vol. 19, 1947.

2) A. Kobori, loc. cit.