4. On the Flat Conformal Differential Geometry, IV.

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(Comm. by S. KAKEYA, M.I.A., Feb. 12, 1946.)

§ 4. *Theory of subspaces.*

We have, in Chapters 1 and 2, established the fundamental differential equations of the flat conformal geometry, and, in Chapter 3, discussed the curves in the flat conformal space and established the Frenet formulae for curves with respect to a projective parameter and with respect to a conformal parameter. In the present Chapter, we shall deal with subspaces in the flat conformal space.

1°. *Subspaces in the flat conformal space*

Let us consider an m-dimensional subspace C_m:

(4.1) $$\xi^\lambda = \xi^\lambda(\xi^{\dot 1}, \xi^{\dot 2}, \ldots\ldots, \xi^{\dot m})$$

in the n-dimensional flat conformal space C_n described by a curvilinear coordinates system (ξ^λ). Then, the current point-hypersphere $A_{\dot 0} = A_0$ on the subspace may be considered as function of m parameters ξ^i $(i, j, k, \ldots\ldots = \dot 1, \dot 2, \ldots\ldots, \dot m)$. Differentiating the relation $A_{\dot 0}A_{\dot 0} = 0$, we know that, the hyperspheres

$$A_i = \frac{\partial A_{\dot 0}}{\partial \xi^i} = \frac{\partial \xi^\lambda}{\partial \xi^i}\frac{\partial A_0}{\partial \xi^\lambda},$$

or

(4.2) $$A_i = B_i^{\cdot\lambda} A_\lambda \qquad \left(B_i^{\cdot\lambda} = \frac{\partial \xi^\lambda}{\partial \xi^i}\right)$$

pass through the point $A_{\dot 0}$. Moreover, since $dA_{\dot 0} = d\xi^i A_i$ along the subspace, and consequently each hypersphere A_i belongs to a pencil of hyperspheres determined by the point $A_{\dot 0}$ and a nearby point $A_{\dot 0} + dA_{\dot 0}$ on the subspace, we see that A_i are m hyperspheres orthogonal to the subspace. From (4.2), we have

(4.3) $$A_j A_k = g_{jk} = B_j^{\cdot\mu} B_k^{\cdot\nu} g_{\mu\nu}.$$

Now, we shall choose $n-m$ mutually orthogonal unit hyperspheres A_P $(P, Q, R, \ldots = \dot m + \dot 1, \ldots, \dot n)$ all passing through the point $A_{\dot 0}$ and tangent to the subspace C_m.

Then the hyperspheres A_P, all passing through the point $A_{\dot 0}$, may be expressed, with respect to the repere $[A_0, A_\lambda, A_\infty]$, in the form

1) K. Yano: On the flat conformal differential geometry, I, II, III. Proc. *21* (1945), 419–429; 454–465; *22* (1946), 9–19.