PAPERS COMMUNICATED:

1. On Canonical Transformations.*)

By Taro URA.

Astronomical Department, Tokyo Imperial University

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§1. *Introduction.*

It is known that a contact transformation does not change the form of a Hamiltonian canonical system of differential equations. Transformations not changing the canonical form are generally called *canonical*. For the transformations to be canonical, however, it is not necessary to be contact. Although extension of the contact transformations has been made,**) it does not include all the canonical transformations. In the present note the necessary and sufficient condition for canonicity is obtained purely algebraically and theorems on canonical transformations especially the necessary and sufficient condition for the existence of linear canonical but not contact transformations for a Hamiltonian canonical system are proved.

Let

$$\dot{q}_i \equiv \frac{d}{dt} q_i = \frac{\partial H}{\partial p_i}, \quad \dot{p}_i \equiv \frac{d}{dt} p_i = -\frac{\partial H}{\partial q_i}, \quad (i = 1, \ldots, n) \tag{1}$$

be a given canonical system, where H is assumed not to contain the variable t explicitly.

We transform the variables q, p to Q, P by the equations

$$Q_i = Q_i(q,p), \quad P_i = P_i(q,p), \quad (i = 1, \ldots, n), \tag{C}$$

and assume that the functions Q, P do not contain t and, for the convenience of explanation, we shall use the matrix-, tensor- and vector-notations (The dummy indices of tensors run over from 1 to n.)

Writing

$$\mathfrak{q} = \begin{pmatrix} q_1 \\ \vdots \\ q_n \end{pmatrix}, \quad \mathfrak{p} = \begin{pmatrix} p_1 \\ \vdots \\ p_n \end{pmatrix}, \quad \mathfrak{Q} = \begin{pmatrix} Q_1 \\ \vdots \\ Q_n \end{pmatrix}, \quad \mathfrak{P} = \begin{pmatrix} P_1 \\ \vdots \\ P_n \end{pmatrix};$$

$$\mathfrak{h}_1 = \begin{pmatrix} \frac{\partial H}{\partial q_1} \\ \vdots \\ \frac{\partial H}{\partial q_n} \end{pmatrix}, \quad \mathfrak{h}_2 = \begin{pmatrix} \frac{\partial H}{\partial p_1} \\ \vdots \\ \frac{\partial H}{\partial p_n} \end{pmatrix}, \quad \mathfrak{H}_1 = \begin{pmatrix} \frac{\partial H}{\partial Q_1} \\ \vdots \\ \frac{\partial H}{\partial Q_n} \end{pmatrix}, \quad \mathfrak{H}_2 = \begin{pmatrix} \frac{\partial H}{\partial P_1} \\ \vdots \\ \frac{\partial H}{\partial P_n} \end{pmatrix}, \tag{2}$$

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** S. Lie, *Die Allgemeinste Berührungstransformationen*, Gesammelte Abhandlungen 3, 295.