62. Note on Irreducible Rings.

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The purpose of the present work(1) is to extend, partly, the well-known beautiful theory of simple algebras and their relationship with subalgebras(2) to irreducible rings; A ring we call *irreducible*, or *right-irreducible* to be precise, when it has a faithful irreducible right-module. More generally we call a ring *(right-)semi-irreducible*, when it has a faithful completely reducible right-module.(3) If an (irreducible) ring possesses a faithful irreducible right-ideal, then we speak of a *(right-) ideal-irreducible* ring. A *closed (right-) irreducible* ring is defined as a ring $\mathfrak{R}$ possessing a faithful irreducible right-module $\mathfrak{m}$ with $\mathfrak{R}$-endomorphism ring $\mathfrak{R}^*$, such that every $\mathfrak{R}^*$-endomorphism of $\mathfrak{m}$ is induced by $\mathfrak{R}$. Similarly defined are *(right-) ideal-semi-irreducible* and *closed (right-) irreducible* rings.

Let $\mathfrak{R}$ be a (right-) ideal-semi-irreducible ring, $\mathfrak{r}_1$ a faithful completely reducible right-ideal in $\mathfrak{R}$. Take one representative from each class of mutually isomorphic irreducible right subideals of $\mathfrak{r}_1$. The (restricted direct) *sum* $\mathfrak{r}_0$ *of the tatality of such representatives* is also a faithful completely reducible right ideal. Now we have:

Every faithful right-module of $\mathfrak{R}$ possesses a submodule isomorphic to $\mathfrak{r}_0$. In particular, $\mathfrak{r}_0$ is a minimal faithful right ideal in $\mathfrak{R}$. Every non-zero right-ideal of $\mathfrak{R}$ contains an irreducible right subideal, which is isomorphic with an irreducible component of $\mathfrak{r}_0$. A right-ideal of $\mathfrak{R}$ is irreducible if and only if it is generated by a primitive idempotent element. The sum of all (irreducible) right-ideals isomorphic with an irreducible right-ideal is an irreducible two-sided ideal, and every irreducible two-sided ideal is obtained in such manner. Every non-zero two-sided ideal contains an irreducible two-sided ideal. The (restricted direct) sum of all irreducible two-sided ideal, that is, the largest completely reducible two-sided ideal in $\mathfrak{R}$, is the smallest right-(as well as two-

(1) A fuller account is given in a forthcoming joint paper by G. Azumaya and the writer.

(2) Of R. Brauer, E. Noether and A. A. Albert, among others.

(3) For C. Chevalley's principal theorem of semi-irreducible ring, in the effect to embed a semi-irreducible ring densely in a closed one (in the sense of the weak topology of mappings in the (discrete) module, see T. Nakayama, Ueber einfache distributive Systeme unendlicher Ränge, these Proc. 20 (1944), Anhang.