61. *New Foundation of the Theory of Simple Rings.*

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Beautiful theory of simple rings and their subrings has been developed mainly by Brauer, Noether and Albert.(1) Jacobson has recently succeeded in obtaining further the Galois theory for quasi fields.(2)

Under a certain new idea we want in the present paper to re-establish and generalize these theories. Our basic method(3) used in the whole consists principally in the fact that if $\mathfrak{R}$ is a simple ring (i.e. a matrix ring over a quasi-field) and $\mathfrak{M}$ a finite $\mathfrak{R}$-module then the $\mathfrak{R}$-endomorphism ring $\mathfrak{R}^*$ of $\mathfrak{M}$ is also simple, $\mathfrak{M}$ is finite with respect to $\mathfrak{R}^*$ and $\mathfrak{R}$ is considered conversely as an $\mathfrak{R}^*$-endomorphism ring of $\mathfrak{M}$; further to every automorphism of $\mathfrak{R}$ there belongs at least one semi-linear transformation of $\mathfrak{M}$. This, together with other related theorems, is discussed in § 1. After these preparations we are able to give in § 2 a quite natural and direct proof to the well-known fundamental theorem for simple rings. In some

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(1) See R. Brauer, Über Systeme hyperkomplexer Zahlen, Math. Zeitschr. 30 (1929); E. Neother, Nichtkommutative Algebra, Math. Zeitschr. 39 (1933); M. Deuring, Algebren, Ergebn. Math. 4 (1935); A. A. Albert, Stucture of Algebras, New York (1939).

(2) N. Jacobson, The fundamental theorem of Galois theory for quasi-fields, Ann. Math. 41 (1940).

(3) It is perhaps of some interest to compare our method with those hitherto given. Brauer first used the algebraic closure of coefficient field. Noether built the theory on her beautiful theory of representations in quasi-fields; the difficulties in separability were so removed and the theory was extended from algebras to rings. Embedding the algebra into matrix algebra over the ground field, Weyl and Brauer avoided the representation theory in quasi-fields, though were restricted again to algebras and the theory was not fully expounded; a complete derivation of the theory along this line was given in a note by Kawada-Oi (Zenkoku-Shijô-Sugaku-Danwakai 162). The similar effect was accomplished in Albert's method by forming the direct product with an inverse-isomorphic algebra; a similar approach was given independently also by Chevalley and Nakayama in their seminary in Princeton, as the writer has been informed. Our method is however to characterize the simple ring as a subring of an absolute endomorphism ring (of a certain module). If we restrict ourselves to algebras then our method is more or less similar to those of Weyl-Brauer and Albert. But our absolute endomorphism ring enables us, not only it is much more natural and directer than the matrix ring over the ground field, to built the theory in a far more general case. Indeed Nakayama, to whom also the present work owes useful remarks, has found that our method is particularly of use in his study of "irreducible rings." (The theory will be reported shortly in these proceedings as well as in a joint paper with Nakayama and the writer.)