

57. *On the Infinitesimal Deformations of Curves in the Spaces with Linear Connection.*

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§0. *Introduction.*

Since T. Levi-Civita⁽¹⁾ has published his famous paper on the geodesic deviation, the theory of infinitesimal deformations of the curves was studied by T. Boggio, E. Bortolotti, E. Cartan, U. Crudeli, E. T. Davies, L. P. Eisenhart, A. De Mira Fernandes, H. A. Hayden, V. Hlavaty, M. S. Knebelman, A. J. McConnell, O. Onicescu, J. A. Schouten, J. L. Synge, G. Vranceanu and others. The theory of infinitesimal deformations of curves was then generalized to that of subspaces by E. Bortolotti, E. Cartan, E. T. Davies, H. A. Hayden, A. J. McConnell, J. A. Schouten, A. G. Walker, C. G. Weatherburn, K. Yano and others. Recently, the theory of infinitesimal deformations of the space itself was studied by N. Coburn, D. v. Dantzig, E. T. Davies, P. Dienes, L. P. Eisenhart, E. R. van Kampen, M. S. Knebelman, A. J. McConnell, J. A. Schouten, W. Slebodzinski, K. Yano and others.

One of the present authors⁽²⁾ has recently developed a geometrical theory of infinitesimal deformations and studied especially the deformations of subspaces imbedded in a space with linear connection.

In the present paper, we shall apply these methods, those of the above mentioned authors and that of K. Yano, to the study of the infinitesimal deformations of curves, particularly, of geodesics, affine conics, projective conics, geodesic circles and conformal circles. We shall state here only the results, the full detail will be published elsewhere.

§1. *The definition of the operator Δ .*

Let us consider a space of n dimensions with an affine connection $\Gamma_{\mu\nu}^{\lambda}$. A curve in the space being represented by the equations of the form $x^{\lambda} = x^{\lambda}(t)$, where t is an arbitrary parameter, we shall consider the infinitesimal deformation

$$(1.1) \quad \bar{x}^{\lambda}(\bar{t}) = x^{\lambda}(t) + \xi^{\lambda}(t) \delta u.$$

(1) T. Levi-Civita: Sur l'écart géodésique. *Math. Ann.*, 97 (1926), 291-320.

(2) K. Yano: Sur la théorie des déformations infinitésimales. To appear in the *Journal of the Faculty of Science, Imperial University of Tokyo.*