48. On Krull's Conjecture Concerning Completely Interally Closed Integrity Domains, III.

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(Comm. by T. TAKAGI, M. I. A., Sept. 12, 1946.)

As was kindly called attension by Mr. G. Azumaya, the argument in the previous parts I, II[1] contained a lack, which the writer wants to make up in the following. It was proved, namely, that the Archimedean vector-lattice $\mathfrak{L} = \mathfrak{L}_\Omega$, constructed in I, can not be faithfully represeneted by (finite) real-valued functions, but the representations considered in the proof there were lattice-representations preserving meet and join, and so it was not shown that it can not be order-isomorphically represented by a group of real-valued functions. However, a slight modification of the proof shows this latter too, and thus our counter-example to Krull-Clifford's problem remains valid.

Let A be, as before, a complete Boolean algebra containing a countable set of non-zero and non-atomic elements $v_1, v_2, \dots, v_i \dots$ such that for any $a > 0$ in A we have $a \geqq v_i$ with a suitable i; we may take as A, for instance, the complete Boolean algebra of regular open sets of the interval (0, 1). Let $\Omega = \Omega(A)$ be its representation space, and let $\mathfrak{L}' = \mathfrak{L}_\Omega$ be the vector-lattice of real- and $\pm\infty$-valued continuous functions on Ω finite except on nowhere-dense sets. Then

Theorem 0. The Archimedean partially ordered (additive) group $\mathfrak{L}$ can never be order-isomorphically represented by (finite) real-valued functions. In fact, it has no non-trivial order-preserving homomorphic mapping into th ordered additive group of real numbers.

Proof. Let $g \to \alpha(g)$ ($g \varepsilon \mathfrak{L}$) be an order- and group-homomorphic mapping of $\mathfrak{L}$ into the ordered group of real numbers. Let $\mathfrak{p}$ be an arbitrary point in Ω. We assert that there exists an element g in $\mathfrak{L}$ such that $g \geqq 0$ (or, what is the same, $g(\mathfrak{q}) \geqq 0$ for every $\mathfrak{q} \varepsilon \Omega$), $g(\mathfrak{p}) \geqq 1$ and $\alpha(g) = 0$. Namely, assume the contrary and suppose that $\alpha(g)$ ($\neq$whence) > 0 whenever $g(\mathfrak{p}) \geqq 1$, $g \geqq 0$. Let $w_1 \geqq w_2 \geqq \dots \geqq w_i \geqq \dots$ be a monotonic sequence of elements in (the maximal prime dual ideal) $\mathfrak{p}$ such that inf $w_i = 0$ (cf. I, Lemma 1). Then w_1-set $\geqq w_2$-set $\geqq \dots \geqq w_i$ -set $\geqq \dots \wedge (w_i$ -set) (no-

(1) Proc. Imp. Acad. Tokyo 18 (1942).