46. On the Unitary Equivalence in Genral Euclid Space.

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I. *Introduction and the theorem.* The problem of the unitary equivalence of two bounded self-adjoint (s. a.) operators in Hilbert space was solved by E. Hellinger[1] and H. Hahn;[2] the result was extended by M. H. Stone[3] to the case of not necessarily bounded s. a. operators. Later, K. Friedrichs[4] and H. Nakano[5] obtained respectively new forms of the condition for the unitary equivalence; and their results were respectively extended by F. Wecken[6] and H. Nakano[7] to the case of general euclid space R (the space in which all the axioms of the Hilbert space are satisfied except the axiom of separability). The purpose of the present note is to give a condition of tne unitary equivalence in a form somewhat more simple and more algebraical than those of the above cited authors. It is easy to see[8] that we may reduce the problem to the case of bounded s. a. operators T_1 and T_2. For any bonnded s. a. operator T let $(T)'$ be he totality of the bounded linear operators commutative with T, and let $(T)''$ be the totality of the bounded linear operators commutative with every operator $\varepsilon (T)'$. Then $(T)'$ and $(T)''$ are operator rings (with complex multipliers) and satisfy the condition

(1) if $S \varepsilon (T)'$ ($(T)''$) the conjugate operator S^* also $\varepsilon (T)'$ ($(T)''$).

Moreover the ring $(T)''$ is commutative. In terms of the operator-ring theory our result reads as follows.

Theorem. For the unitary equivalence of T_1 and T_2 it is necessary and sufficient that the ring $(T_1)'$ is isomorphic (with complex multipliers) to the ring $(T_2)'$ by a correspondence C which maps T_1 onto T_2 and which maps conjugate operators onto conjugate operators.

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(1) Dissertation, Göttingen' 1907.
(2) Monatsheft Math. u. Phys. *23* (1912), 169–224.
(3) Linear transformations in Hilbert space, New York 1932.
(4) Jahresber. d D. Math. Ver. *45* (1935) II, 79–82.
(5) Ann. of Math. *42* (1941), 657–664.
(6) Math. Ann. *116* (1939), 422–455.
(7) Math. Ann. 118 (1941), 112–133.
(8) Consider $\mathrm{Tan}^{-1} T_1$ and $\mathrm{Tan}^{-1} T_2$ if T_1 and T_2 are unbounded.