44. On the Singularities of Analytic Functions with a General Domain of Existence.

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1. Recently S. Kametani and M. Tsuji have investigated the behaviour of a meromorphic function with the set of capacity zero of essential singularities.(1) Let E be a bounded closed set of capacity zero. Suppose that $w = w(z)$ is uniform and meromorphic outside E and has a transcendental singularity at every point of E. Tsuji has found that a theorem of Evans(2) plays an important rôle in such an investigation and obtained systematically some interesting theorems concerning the behaviour of $w = w(z)$. Evans' theorem states that there exists a distribution of positive mass $d\mu(a)$ entirely on E such that

$$(1) \qquad u(z) = \int_E \log\left|\frac{1}{z-a}\right| d\mu(a), \int_E d\mu(a) = 1$$

tends to $+\infty$, when z tends to any point of E. Let $v(z)$ be its conjugate harmonic function and put

$$(2) \qquad \zeta = \chi(z) = e^{u(z)+iv(z)} = \rho(z)\, e^{iv(z)}.$$

Let C_r be the niveau curve: $\rho(z) = \text{const.} = r$, then C_r consists of a finite number of simple closed curves surrounding E and moreover there holds

$$(3) \qquad \int_{C_r} dv(z) = \int_{C_r} \frac{\partial u}{\partial n} ds = 2\pi,$$

where ds is the arc length of C_r and n is the inner normal of C_r. Suggested from Tsuji's proof for the extension of Gross'(3) theorem concerning the principal star-region of an inverse element of $w = w(z)$, the present author

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