# 6. *Finite Groups with Faithful Irreducible and Directly Indecomposable Modular Representations.*

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The structure of finite groups possessing faithful (isomorphic) irreducible representation (i. r.) in a non-modular field has been determined by K. Shoda[1]; his argument was supplemented by Y. Akizuki. The result is: A finite group $\mathfrak{G}$ possesses a faithful i. r. in an (arbitrary) non-modular field, if and only if for every ideal factor of the product of all minimal abelian invariant subgroups the inequality

$$\text{(S)} \qquad c \leqq m/\lambda$$

is fulfilled, where $c$, $l^m$ and $l^\lambda$ denote respectively the number of minimal factors in the ideal factor, the order of the minimal factor, and the number of elements in the $\mathfrak{G}$-automorphism quasifield of the minimal factor.

A somewhat generalized problem to determine those finite groups which have faithful non-modular representations with $t$ irreducible components (i. c.), where $t$ is a natural number, has been considered by M. Tazawa[2]. The result is to replace (S) by

$$\text{(T)} \qquad c-[(t-1)c/t] \leqq m/\lambda$$

Now in the present note we consider modular representations[3]. Here i. r., directly indecomposable representations (d. i. r.) and directly indecomposable components of regular representation (d. i. c. of r. r.) are three classes of representation of particular concern. Our results about faithful representations of these kinds are similar to the above theorems of Shoda and Tazawa, and assume more or less expected forms. Namely:

*Theorem 1. Let K be an arbitrary field of charactristic $p$, and $\mathfrak{M}$ the product of all abelian minimal subgroups* of order prime to $p$ *in a finite group $\mathfrak{G}$. Then:* i) *$\mathfrak{G}$ possesses a faithful d. i. r. (resp. representation with t directly indecomposable components (d. i. c.)) in K if and only if* (S) (*resp.* T)) *is satisfied for every ideal factor in $\mathfrak{M}$*; ii) *The same is also necessary and sufficient in order that $\mathfrak{G}$ have a faithful d.i.c. of r. r. (resp. representation decomposed (directly) into t d. i. c. of r. r.) in K;* iii) *$\mathfrak{G}$ has a faithful i. r. (resp. completely reducible representation*

1) K. Shoda, Über direkt zerlegbare Gruppen, Journ. Fac. Sci. Tokyo Imp. Univ. Section I, Vol. II-3 (1930), § 7; correction, Vol. II-7 (1931).

2) M. Tazawa, Über die isomorphe Darstellung der endlichen Gruppe, Tohoku Math. J. 47 (1940).

3) For modular representations of finite groups, in particular the theory of Brauer-Nesbitt, see the references in M. Osima, 有限群のモヂュラー表現, 日本數學物理學會誌 16 (1942); in the following we shall not however need any deeper part of the theory.