1. *Fundamental Differential Equations in the Theory of Conformal Mapping.*

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1. Let $\mathfrak{S}$ be the family of analytic functions $F(z)$ which are regular and schlicht in the interior of the unit circle E: $|z|<1$ and further are normalized at the origin in such a way that $F(0)=0$, $F'(0)=1$. The theory of this family has been developed by various methods. Among them, one based upon the so-called Löwner's differential equation[1] on bounded slit mapping of E has been, especially first by Sovietic mathematicians G. M. Golusin, J. Basilewitsch etc., shown to be very fruitful. Let B be a bounded slit domain obtained from $|w|<1$ by cutting it along a Jordan arc which lies in $|w|<1$ save an end-point and does not pass through the origin. The mapping function $w=f(z)$, $f(0)=0$, $f'(0)=e^{-t_0}$, of E onto B is then regarded as the integral $f(z)=f(z, t_0)$ of the so-called Löwner's differential equation.

$$\frac{\partial f(z,t)}{\partial t}=-f(z,t)\frac{1+\kappa(t)f(z,t)}{1-\kappa(t)f(z,t)} \qquad (0\leqq t\leqq t_0) \tag{1.1}$$

with initial condition $f(z, 0)=z$, $\kappa(t)$ being a continuons function weose absolute value is identically equal to unity. Each function $w_t=f(z, t)$, for which $f(0, t)=0$ and $f'(0, t)=e^{-t}$, gives also a bounded slit mapping of E. Introduce now a new family of slit mapping functions $\{h(z, t)\}$ $(0\leqq t\leqq t_0)$ by functional relation

$$f(z)=h(f(z,t),t). \tag{1.2}$$

Then the differential equation for this family becomes

$$\frac{\partial h(z,t)}{\partial t}=z\frac{1+\kappa(t)z}{1-\kappa(t)z}\,\frac{\partial h(z,t)}{\partial z} \qquad (t_0\geqq t\geqq 0) \tag{1.3}$$

with boundary conditions $h(z, t_0)=z$ and $h(z, 0)=f(z)$.

Now, remembering the structure of Löwner's differereutial equation, we may expect that analogous equations can be constructed in various ways from more general point of view. We consider, in general, a function $w=F(z)$ which maps E onto a given simply connected domain D in the w-plane. Suppose that a family of simply connected domains $\{D_t\}$ with a real parameter t $(0\leqq t\leqq t_0)$ be constructed in such a way that D_0 and D_{t_0} coïncide with the domains $|w|<1$ and D

1) K. Löwner, Untersuchungen über schlichte konforme Abbildungen des Einheitskreises, I. Math. Ann. **89** (1923), 103–121.