17. *Fundamental Theory of Toothed Gearing (II).*

By Kaneo YAMADA.
Department of Applied Dynamics, Tôhoku Univer i y, Sendai.
(Comm. by T. KUBOTA, M.J.A., Feb. 12, 1949.)

Assume that a curve K is oriented to a certain direction. Take any two points P_0 and P on K. We say that the arc length P_0P is positive or negative according as P exists at the positive side or negative side of P_0. The orientation of the tangent to K at any point may be defined in usual manner in accordance with that of K. Let C be an arbitrary point. We give the length of the segment PC of the straight line connecting P with C a positive or negative sign according as C exists on the left side or right side of the tangent to K at P. Referring to a pair of pitch curves K_1 and K_2 we shall assume that they are oriented in the same sense, that is, the common tangent at every instant has the same sense even if observed as a tangent of K_1 or of K_2.

§ 1. Analytical representation of profile curves.

Given an oriented curve K and determined on it an arbitrary point P_0 as origin, then we can indicate the position of any point P on K by the arc length P_0P which we shall denote by s and call the abscissa of P on K. Now consider a family of circles with centers on K. This family is established when the following relation is given:

$$(1) \qquad r=f(s)$$

between the abscissa s of any point P on K and the radius r of the circle with P as the center. If this family possesses envelopes, we can determine one F of them, when the sign of r in Equation (1) is indicated. In this case $|f(s)|$ is a one-valued continuous function of s and we may assume further it is differentiable as regards s such that $|f(s)| \gtrless 0$. Next, we denote by θ the angle between the perpendicular drawn from an arbitrary point P on K to the curve F and the oriented tangent to K at P. We shall also give θ the same sign as that of r. Then we have the following simple relation among these three quantities r, s and θ:

$$(2) \qquad \frac{d|r|}{ds}=-\cos\theta\,, \quad sgn(\theta)=sgn(r)\,.$$

Now we shall conclude the proof of Theorem 2 in the report (I). The pair of envelopes F_1 and F_2 which we already determined in the first half of the proof