16. *Fundamental Theory of Toothed Gearing (I).*

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(Comm. by T. KUBOTA, M.J.A., Feb. 12, 1949.)

Suppose that two plane curves K_1 and K_2 roll without sliding mutually one along the other, and let F_1 and F_2 which roll, in accordance with the rolling motion of K_1 and K_2, with sliding mutually one along the other be the two plane curves invariably connected with K_1 and K_2 respectively. K_1 and K_2 are called the corresponding pitch curves, and F_1 and F_2 the corresponding profile curves. Furthermore, we shall call any two points of K_1 and K_2 which may fall on each other at the rolling motion of K_1 and K_2 the corresponding pitch points, and especially a point at which K_1 and K_2 are touching at a certain instant the common pitch point at the instant.

From now on we confine ourselves to deal with such continuous (pitch or profile) curves as at each of points on them a single tangent may be drawn continuously (although cusps are allowed to exist), and suppose that they touch each other always at one point during the motion.

§ 1. Necessary and sufficient conditions for profile curves (1).

As a necessary condition that two curves F_1 and F_2 invariably connected with two pitch curves K_1 and K_2 respectively be a pair of profile curves the following Descartes' theorem (a) is well known.

(a) *The common normal to the curves F_1 and F_2 at any point of contact of them always passes through a common pitch point*

From the condition (a) we obtain the following necessary and sufficient condition for profile curves.

Theorem 1. *A necessary and sufficient condition that two curves F_1 and F_2 invariably connected with two pitch curves K_1 and K_2 respectively be a pair of profile curves is that two perpendiculars from any common pitch point to F_1 and F_2 coincide with each other in the direction and in the length to their feet.*

As a necessary condition that a curve F settled at one K of pitch curves K_1 and K_2 be a profile curve, that is, there exists a corresponding curve to F which makes sliding contact motion with F, we can derive directly the following condition (b) from the condition (a).

(b) *The curve F is an envelope of a family of circles, each of which*