23. *On the Potential Defined in a Domain.*

By Tadao KUBO.
Tokushima Technical College.

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Let us consider a simply connected "schlicht" domain R on the z-plane whose boundary is a simple closed Jordan curve and an additive class F composed of the sets of points contained in a bounded closed subset E of R.

We suppose that a function μ ($\geqq 0$) of the sets is completely additive with respect to any set belonging to F.

Then we shall define the potential of mass-distribution μ on E in the form

(1) $$V(z)=\int_E g(z,\zeta)d\mu(\zeta),$$

where g (z, ζ) is a Green's function of the domain R with a pole ζ and z is any fixed point in R.

The integral(1) has a meaning in the sense of the Stieltjes Lebesgue-Radon's integral.

From the definition(1), we easily obtain

$$\varDelta V(z)=0 \qquad (\varDelta \text{ is Laplacian})$$

at any point in the free space R-E, for $\varDelta g(z,\zeta)=0$.

Now we shall study whether Gauss' theorems[1] on the potential in the usual sense hold for the potentil (1) in our definition, succeeding to the idea of "Green's Geometry"[2] discussed by Prof. Matsumoto.

Let the subset E be lying entirely in R. Then we can suitably choose a constant c (>0) such that the subset E is entirely enclosed by the equipotential curve C_0 : $g(z,z_0)=c$ of Green's function of R with a pole z_0

Thus, let us consider the arithmetic mean of the potential (1) by integration on C_0 for which we shall use the non-Euclidean (hyperbolic) metric $d\sigma_z$[3] for the linear element.

Such an arithmetic mean by integration, we denote by $A\{V_{(z)}\}$ for simplicity.

By Fubini's theorem on the change of order of integration, we have

(2) $$\int_{C_0} V(z)\,d\sigma_z=\int_E\left(\int_{C_0} g(z,\zeta)d\sigma_z\right)d\mu(\zeta)$$

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1) O. D. Kellogg : Foundations of Potential Theory (1929) P. 82.
2) T. Matsumoto : Gekkan 'Sugaku' October, November, (1937).
3) R. Nevanlinna : Eindeutige Analytische Funktionen (1936) S. 48.